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Final Report, Vol. 6

THE DEVELOPMENT OF OPTIMAL STRATEGIES FOR MAINTENANCE, REHABILITATION AND REPLACEMENT OF HIGHWAY BRIDGES, FINAL REPORT VOL. 6: PERFORMANCE ANALYSIS AND OPTIMIZATION

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FINAL REPORT

The Development of Optimal Strategies for Maintenance Rehabilitation and Replacement of Highway Bridges, Final Report Vol. 6: Bridge Performance and Optimization

TO:	Harold L. Michael, Director Joint Highway Research Project	August 15, 1989 Revised October 5, 1990 Project: C-36-73I
FROM:	Kumares C. Sinha, Research Engineer Joint Highway Research Project	File: 3-4-10

Attached is the Vol. 6 of the Final Report on the HPR Part II Study entitled, "The Development of Optimal Strategies for Maintenance Rehabilitation and Replacement of Highway Bridges." This volume provides the results of the research conducted on the development of an optimization model for bridge project selection. Also included is a discussion on bridge condition deterioration curves and appropriate performance prediction models. The volume was prepared by Yi Jiang under my direction.

This report is forwarded for review, comment and acceptance by the INDOT and FHWA as partial fulfillment of the objectives of the research.

Respectfully submitted,

K. C. Sinha
Research Engineer

KCS/rrp

cc: A.G. Altschaeffl, J.L. Chameau, W.F. Chen, W.L. Dolch, A.A. Fendrick, J.D. Fricker, D.W. Halpin, G.L. Henneke, K.R. Hoover, C.W. Letts, C.W. Lovell, D.W. Lucas, J.F. McLaughlin, R.D. Miles, D.C. Nelson, P.L. Owens, B.K. Partridge, C.F. Scholer, G.B. Shoener, K.C. Sinha, C.A. Venable, T.D. White, L.E. Wood

FINAL REPORT

The Development of Optimal Strategies for Maintenance, Rehabilitation and Replacement of Highway Bridges, Final Report Vol 6: Performance Analysis and Optimization

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16. Abstract

This volume is the sixth of a six-volume final report and it presents the results of the research on bridge performance analyses and the development of an optimization model for bridge project selection.

The tiles of all six volumes are listed below:

Vol. 1. Elements of Indiana Bridge Management System

Vol. 2. A System for Bridge Structural Condition Assessment

Vol. 3. Bridge Traffic Safety Evaluation

Vol. 4. Cost Analysis

Vol. 5. Priority Ranking Method

Vol. 6. Bridge Performance and Optimization

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CHAPTER 1: INTRODUCTION

1.1 Background

It is estimated that $50 billion would be needed to replace or rehabilitate the deficient bridge in this country [FHWA 1987a]. The major elements of the bridge problem are aging and obsolescence. About one-half of the approximately 600,000 highway bridges in the U. S. were built before 1940 [Reilly 1984]. In 1985, seventy five percent of all bridges were reported to be older than the typical 50-year design life for bridges [ITE 1985]. Most of these bridges were designed for less traffic, smaller vehicles, slower speed, and lighter loads than the standards employed for recently built bridges. The Federal Highway Administration (FHWA) recently rated about 45 percent of the existing bridges as either functionally or structurally deficient.

The State of Indiana has a large number of bridges that need immediate attention. There were 5,290 bridges on the state highway system in Indiana in 1985 [IDOH 1985], of which 1,789 bridges - or 34% - were functionally obsolete and 472 bridges - 9% - were rated as structurally deficient. These statistics clearly indicate bridge improvement problems that the State of Indiana will face in the near future.

Faced with budget constraints and the extensive bridge repair and replacement needs, decision makers need an efficient tool for selecting bridge projects among many alternatives. At the network level of decision-making, a comprehensive system would be such a tool. A major objective of a bridge management system is to assist bridge managers in making consistent and cost-

effective decisions related to maintenance, rehabilitation and replacement of bridges on a system wide basis. This systematized approach for making bridge programming decisions is different from applying engineering expertise on a case-by-case basis. In reality, the available budget and the bridge repair and replacement needs are always imbalance. Therefore, a wide variety of tradeoffs come into play. Some projects can be delayed in order to immediately construct a more worthy project, and so on. A quantitative tool is necessary that can assist in evaluating the possible tradeoffs.

1.2 Purpose and Scope of the Study

As part of the effort to develop a comprehensive bridge management system for the Indiana Department of Transportation (INDOT), the main objective of this study was to construct models for bridge condition prediction and optimal bridge project selection. A decision making, either at the network level or at the project level, is based on bridge conditions at present and in the future. It is essential for a bridge management system to have the capability of accurately predicting future bridge conditions.

The research reported in this volume consisted of two parts:

1. bridge performance analysis;
2. development of dynamic optimization model.

The performance analysis provided a prediction model that can be used to predict the future bridge condition rating. The prediction model was developed using the Markov chain to reflect the stochastic nature of bridge condition changes. A dynamic optimization model was developed to optimize bridge project selections. The model applies dynamic programming and integer programming to

select projects while the effectiveness or benefit of a bridge system is maximized subject to the constraints of available budgets over a given program period. The bridge condition prediction model was incorporated in the optimization model. Another version of the project selection model was also developed so that the ranking model discussed in Volume 5 could be incorporated in the optimization model.

1.3 Report Organization

This volume contains five chapters. Chapter 2 presents the results of the bridge condition prediction model. Chapter 3 deals with the dynamic optimization model for project selection. Chapter 4 presents a model that combines the priority ranking and optimization techniques into one interacting model. The summary and conclusions are given in Chapter 5.

CHAPTER 2: BRIDGE PERFORMANCE ANALYSIS

2.1 Introduction

A bridge performance analysis was performed. Performance functions for deck, superstructure and substructure of bridges were developed using regression method. A bridge performance prediction model was also developed using the Markov chain.

Performance function is the relationship between bridge condition rating and bridge age, which reflects the level of service of a bridge, and therefore, is used as a measure of effectiveness in bridge management system. The bridge performance prediction model used the Markov chain, a probability-based method, to reflect the stochastic nature of bridge conditions. The model can be used to predict condition rating of a bridge at a given age.

This study used the techniques of regression, the Markov chain, non-linear programming and combination of those techniques to analyze bridge performances. The results exhibited the power of those techniques, particularly of the Markov chain approach in prediction or estimation of future bridge conditions. The procedure, although simple, was found to provide a high level of accuracy in predicting bridge conditions. This chapter provides detailed descriptions of the development of performance functions and the Markov chain prediction model.

2.2 Data Base and Factor Classifications

All federally supported bridges have been inspected every two years beginning in 1978. The inspection includes ratings of individual components such as deck, superstructure and substructure as well as of the overall bridge condition. According to the FHWA bridge rating system, bridge inspectors use a range from 0 to 9, with 9 being the maximum rating number for the condition of a new bridge [FHWA 1978, 1979].

The complete data base included about 5,700 state owned bridges in Indiana. The bridge data of ADT, rating and bridge age were used for this study. To evaluate the effects of other factors, such as bridge type, climatic region and highway type, bridges were divided into subgroups as shown in Table 2.1.

The data base showed that the bridges on interstate highways carry higher traffic volumes and are in better conditions as compared with those on primary and secondary highways. To reflect this difference, two highway system types, interstate and other state highways, were used.

ADT of bridges on other state highways was grouped into three levels: low (ADT $<$ 5,000), medium (5,000 $\leq$ ADT $<$ 10,000) and high (10,000 $\leq$ ADT). Very few bridges on interstate highways carry less than 5,000 ADT, therefore, ADT on interstate highway bridges was grouped into two levels: low (ADT $<$ 10,000) and high (10,000 $\leq$ ADT).

Climatic conditions may affect bridge performance. In order to study the effect of climate, the area of Indiana was divided into two regions, northern and

Table 2.1 Classification Factors for Bridge Performance Analysis

A. Highway System

1. Interstate Highways
2. Other State Highways

B. Traffic Volume (ADT)

For Interstates:

1. Low Average Daily Traffic (ADT $<$ 10,000)
2. High Average Daily Traffic (10,000 $\leq$ ADT)

For Other State Highways:

1. Low Average Daily Traffic (ADT $<$ 5,000)
2. Medium Average Daily Traffic (5,000 $\leq$ ADT $<$ 10,000)
3. High Average Daily Traffic (10,000 $\leq$ ADT)

C. Climatic Region

1. Northern Region
2. Southern Region

D. Bridge Type

1. Concrete Bridges
2. Steel Bridges

southern regions, as defined in the Indiana Cost Allocation Study [Sinha et al. 1984]. Similarly, two bridge types, concrete and steel bridges, were analyzed separately.

For each combination of ADT level and other factors, 50 bridges were randomly selected and their available data were used for the analysis. Since there were 20 combinations of ADT level and other factors, 1,000 bridges were selected from the data base. The data included average daily traffic (ADT), bridge age, location, bridge type, highway type and condition rating of bridge and bridge components.

2.3 Development of Performance Functions

The objective of developing performance curves was to find the relationship between condition rating and bridge age. A third order polynomial model was used to obtain the regression function of the relationship. The polynomial model is expressed by the following formula [Neter et al. 1985].

$$Y_i(T) = \beta_0 + \beta_1 T_i + \beta_2 T_i^2 + \beta_3 T_i^3 + \varepsilon_i \qquad (2.1)$$

where, $Y_i(T)$ is the condition rating of bridge i at age T, T_i is the bridge age, and ϵ_i is the error term. This equation indicates that the condition rating of a bridge, $Y_i(T)$, depends on the bridge age, T_i.

The SAS statistical package was used to conduct the polynomial regressions [SAS 1985]. The procedure of General Linear Model (GLM) of the SAS package was selected for this analysis. The GLM procedure uses the method of least squares to fit general linear models. It can handle classification variables, either discrete or continuous, to measure quantities [SAS 1985].

Table 2.2 Results of Homogeneity-of-Slopes Tests (Interstate Highways)

			T*ADT		T*T*ADT		T*T*T*ADT	
			F^*	P	F^*	P	F^*	P
Steel	ADT	Dk	0.17	0.68	0.05	0.82	0.71	0.40
		Sp	0.25	0.61	0.85	0.35	1.51	0.22
		Sb	0.14	0.71	1.35	0.25	2.87	0.09
	Cli.	Dk	3.69	0.06	2.40	0.12	1.36	0.24
		Sp	2.05	0.15	1.54	0.21	0.96	0.33
		Sb	0.39	0.53	0.22	0.64	0.16	0.69
Conc.	ADT	Dk	2.14	0.15	3.38	0.07	4.49	0.04
		Sp	1.84	0.17	3.64	0.06	5.25	0.02
		Sb	1.62	0.21	2.92	0.09	4.09	0.05
	Cli.	Dk	0.72	0.40	0.66	0.42	0.82	0.37
		Sp	0.05	0.83	0.06	0.80	0.20	0.65
		Sb	0.48	0.49	0.61	0.44	0.97	0.33

Note: Sample Size = 200;
F^* -- Computed F Value;
P -- P Value or Level of Significance;
Conc. -- Concrete;
Cli. -- Climate;
Dk -- Deck;
Sp -- Superstructure;
Sb -- Substructure.

Table 2.3 Results of Homogeneity-of-Slopes Tests (Other Highways)

			T*ADT		T*T*ADT		T*T*T*ADT	
			F^*	P	F^*	P	F^*	P
Steel	ADT	Dk	0.08	0.93	0.08	0.93	0.14	0.87
		Sp	2.22	0.11	2.67	0.07	2.48	0.09
		Sb	1.22	0.30	0.53	0.59	0.23	0.80
	Cli.	Dk	1.41	0.24	2.73	0.10	3.44	0.07
		Sp	2.46	0.12	3.49	0.06	3.93	0.05
		Sb	0.81	0.37	1.20	0.27	1.16	0.28
Conc.	ADT	Dk	1.64	0.20	0.80	0.45	0.95	0.40
		Sp	1.40	0.25	1.00	0.37	1.39	0.25
		Sb	1.59	0.21	1.92	0.15	2.87	0.06
	Cli.	Dk	1.63	0.20	3.24	0.07	5.09	0.03
		Sp	0.29	0.59	0.92	0.34	1.95	0.16
		Sb	0.92	0.34	1.84	0.18	3.07	0.05

Note: Sample Size = 300;
F^* -- Computed F Value;
P -- P Value or Level of Significance;
Conc. -- Concrete;
Cli. -- Climate;
Dk -- Deck;
Sp -- Superstructure;
Sb -- Substructure.

indicated that for most of the bridge subgroups the effects of ADT and climate were not significant at α=0.05, and only a few of the subgroups had P-values less than 0.05 (but greater than 0.02) for the effects of T*T*T*ADT and T*T*T*Climate. However, because the estimations of β_3's were relatively small and close to zero, and the difference between the values of performance functions using different β_3's was practically not significant, it was concluded that the effects of ADT and climate on bridge performance were not significant. That is, the performance curves remained the same as ADT or climatic region changed. However, the underlying performance data bases for bridges on interstate highways and on other state highways were significantly different. On interstate highways, the bridges were 35 years or less old and most of the ratings were not below 5. On other state highways, bridge ages were in the range from 0 to 60 years and the ratings were in the range from 3 to 9. Due to the different characteristics of bridges on the two highway system types, the performance curves for the two highway types were separately developed.

Since the effects of ADT and climate were not significant according to the statistical analyses, the subgroups of these effects were combined together. Consequently, the final category included two types of bridges, steel bridge and concrete bridge, and two types of highways, interstate highway and other state highway. For a new bridge (age 0), the recorded condition rating was found always to be 9, therefore, β_0 was specified to be 9 in order to make the intercept of the regression line integer and meaningful in practice. The GLM model provided regression functions for the final twelve subgroups. Table 2.4 presents the estimations of regression parameters and R^2 values of these polynomial regressions. For each type of bridges, there were one performance curve for deck, one for superstructure and one for substructure. Figures 2.1

Table 2.4 Estimations of Regression Coefficients

Interstate		β_1	β_2	β_3	R^2
Steel	Deck	-0.41141790	0.02116563	-0.00040387	0.65
	Sup.	-0.45572206	0.02399958	-0.00044201	0.54
	Sub.	-0.44818105	0.02555900	-0.00049875	0.50
Conc.	Deck	-0.36622617	0.01659520	-0.00017162	0.51
	Sup.	-0.34704791	0.01598966	-0.00027160	0.58
	Sub.	-0.34508455	0.01575857	-0.00026681	0.57
Other		β_1	β_2	β_3	R^2
Steel	Deck	-0.34979283	0.01036093	-0.00011009	0.53
	Sup.	-0.34616183	0.01088174	-0.00011870	0.52
	Sub.	-0.34059831	0.01093574	-0.00011953	0.45
Conc.	Deck	-0.30199933	0.00915111	-0.00009409	0.56
	Sup.	-0.29095931	0.00860726	-0.00008815	0.55
	Sub.	-0.31267496	0.00961677	-0.00009876	0.51

Note:

Conc. -- Concrete;
Sup. -- Superstructure;
Sub. -- Substructure.

through 2.4 show the performance curves and actual data points of the regressions.

The trend of the predicted performance curve matched well the actual bridge condition data. The results indicated that bridge component ratings dropped fast at the beginning of a bridge's life, then became more stable as the bridge age increased and dropped fast again after the component condition rating reached 5 or less. It should be noted that bridge condition ratings are subjective judgments of bridge inspectors and thus the trend may reflect inherent human bias. For example, bridge inspectors are generally reluctant to rate a condition 'perfect' after the first initial year and also they tend to consider the condition as rapidly deteriorating after the rating has reached 5.

2.4 Markov Chain Approach

The Markov chain as applied to bridge performance prediction is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition changing from one state to another. These probabilities are represented in a matrix form that is called the transition probability matrix or simply, transition matrix, of the Markov chain. Knowing the present state of bridges, or the initial state, the future conditions can be predicted through multiplications of initial state vector and the transition probability matrix.

According to the FHWA bridge rating system, bridge inspectors use a range from 0 to 9, with 9 being the maximum rating number for a near-perfect condition [FHWA 1979]. Ten bridge condition ratings are defined as ten states with each condition rating corresponding to one of the states. For example, condition

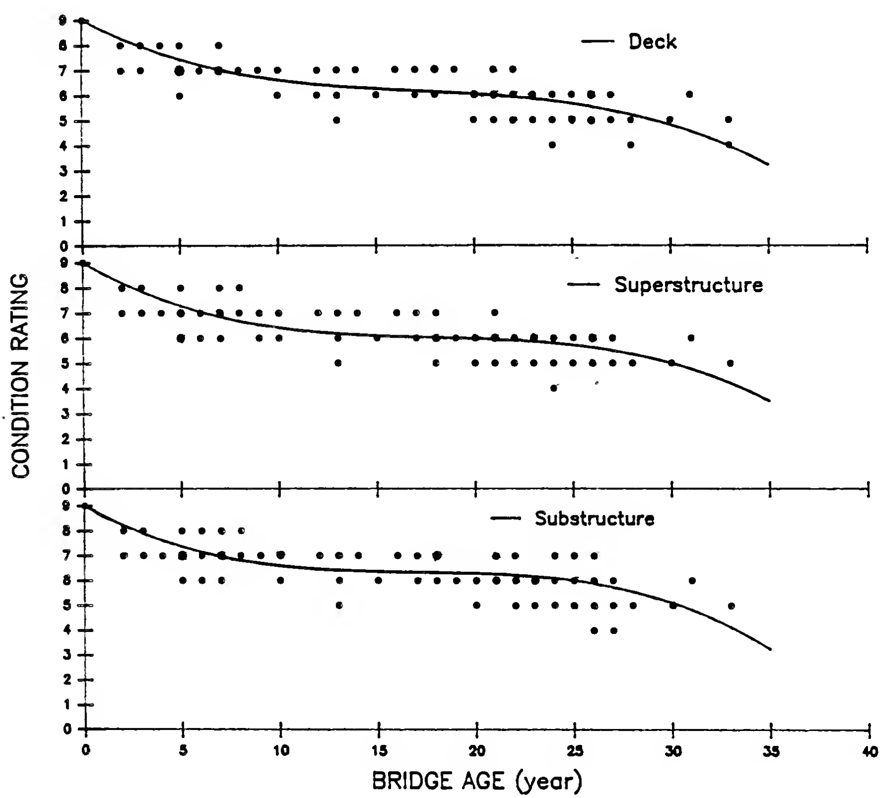

Figure 2.1 Performance Curves of Concrete Bridge Components on Interstate Highways

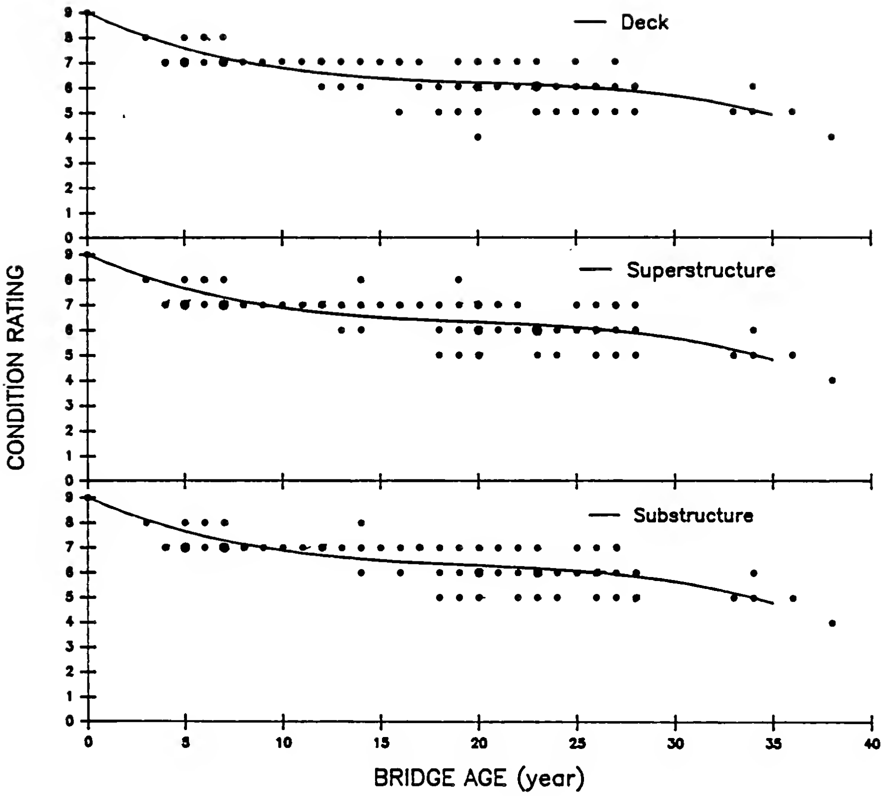

Figure 2.2 Performance Curves of Steel Bridge Components on Interstate Highways

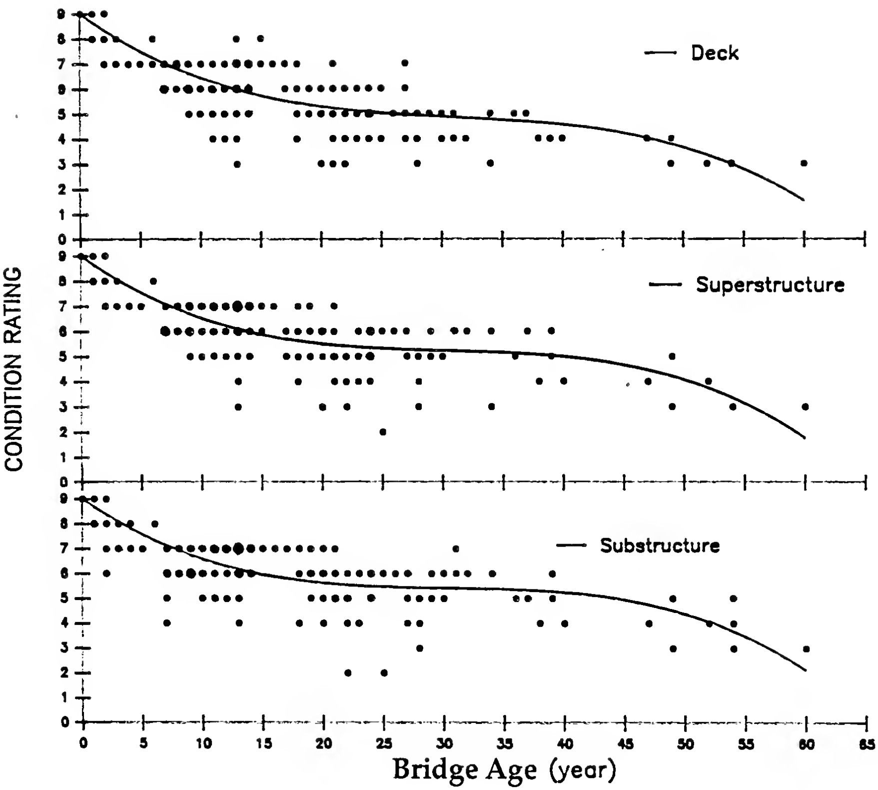

Figure 2.3 Performance Curves of Concrete Bridge Components on Other State Highways

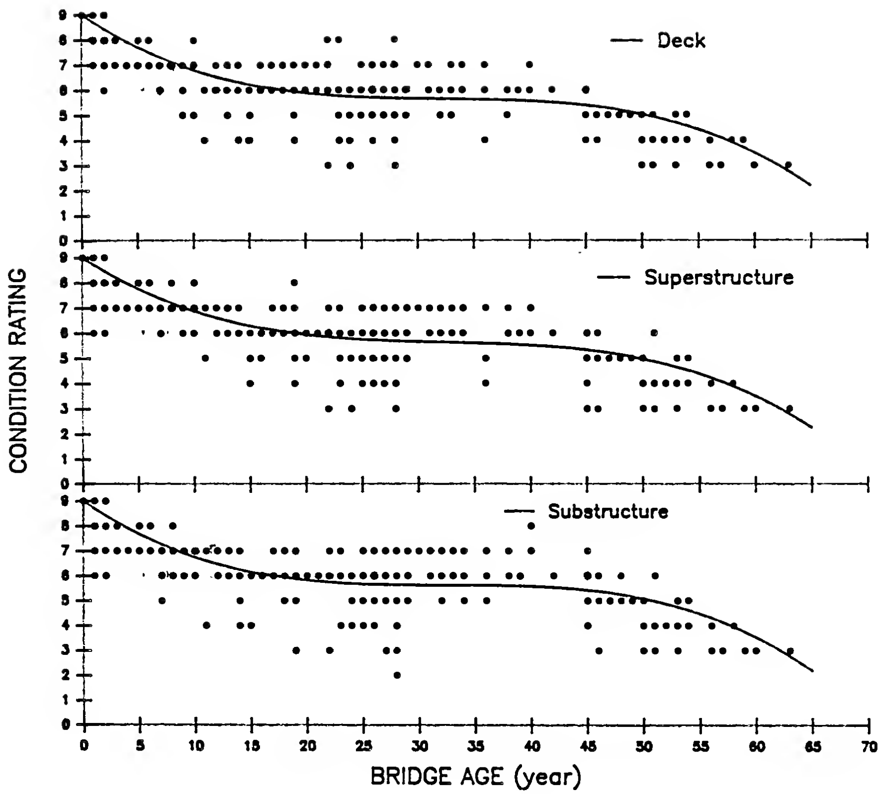

Figure 2.4 Performance Curves of Steel Bridge Components on Other State Highways

rating 9 is defined as state 1, rating 8 as state 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability of condition changing from one state, say i, to another state, j, during a given period of time, which is denoted by $p_{i,j}$. Table 2.5 shows the correspondence of condition ratings, states and transition probabilities.

Let the transition probability matrix of the Markov chain be P, given by

$$p = \begin{vmatrix} p_{1,1} & p_{1,2} & \cdot & \cdot & \cdot & p_{1,10} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{10,1} & p_{10,2} & \cdot & \cdot & \cdot & p_{10,10} \end{vmatrix} \tag{2.3}$$

Then the state vector for any time T, $Q_{(T)}$, can be obtained by the multiplication of initial state vector $Q_{(0)}$ and the Tth power of the transition probability matrix P:

$$Q_{(T)} = Q_{(0)} * P * P * \ldots * P = Q_{(0)} * P^T \tag{2.4}$$

Thus, a Markov chain is completely specified when its transition matrix P and the initial state vector $Q_{(0)}$ are known. Since the initial state vector $Q_{(0)}$ is usually known for a bridge management system, the main problem of the Markov chain approach in this study is to determine the transition probability matrix.

Table 2.5 Correspondence of Condition Ratings, States and Transition Probabilities

		R=9	R=8	R=7	R=6	R=5	R=4	R=3	R=2	R=1	R=0
		S=1	S=2	S=3	S=4	S=5	S=6	S=7	S=8	S=9	S=10
R=9	S=1	$p_{1,1}$	$p_{1,2}$	$p_{1,3}$	$p_{1,4}$	$p_{1,5}$	$p_{1,6}$	$p_{1,7}$	$p_{1,8}$	$p_{1,9}$	$p_{1,10}$
R=8	S=2	$p_{2,1}$	$p_{2,2}$	$p_{2,3}$	$p_{2,4}$	$p_{2,5}$	$p_{2,6}$	$p_{2,7}$	$p_{2,8}$	$p_{2,9}$	$p_{2,10}$
R=7	S=3	$p_{3,1}$	$p_{3,2}$	$p_{3,3}$	$p_{3,4}$	$p_{3,5}$	$p_{3,6}$	$p_{3,7}$	$p_{3,8}$	$p_{3,9}$	$p_{3,10}$
R=6	S=4	$p_{4,1}$	$p_{4,2}$	$p_{4,3}$	$p_{4,4}$	$p_{4,5}$	$p_{4,6}$	$p_{4,7}$	$p_{4,8}$	$p_{4,9}$	$p_{4,10}$
R=5	S=5	$p_{5,1}$	$p_{5,2}$	$p_{5,3}$	$p_{5,4}$	$p_{5,5}$	$p_{5,6}$	$p_{5,7}$	$p_{5,8}$	$p_{5,9}$	$p_{5,10}$
R=4	S=6	$p_{6,1}$	$p_{6,2}$	$p_{6,3}$	$p_{6,4}$	$p_{6,5}$	$p_{6,6}$	$p_{6,7}$	$p_{6,8}$	$p_{6,9}$	$p_{6,10}$
R=3	S=7	$p_{7,1}$	$p_{7,2}$	$p_{7,3}$	$p_{7,4}$	$p_{7,5}$	$p_{7,6}$	$p_{7,7}$	$p_{7,8}$	$p_{7,9}$	$p_{7,10}$
R=2	S=8	$p_{8,1}$	$p_{8,2}$	$p_{8,3}$	$p_{8,4}$	$p_{8,5}$	$p_{8,6}$	$p_{8,7}$	$p_{8,8}$	$p_{8,9}$	$p_{8,10}$
R=1	S=9	$p_{9,1}$	$p_{9,2}$	$p_{9,3}$	$p_{9,4}$	$p_{9,5}$	$p_{9,6}$	$p_{9,7}$	$p_{9,8}$	$p_{9,9}$	$p_{9,10}$
R=0	S=10	$p_{10,1}$	$p_{10,2}$	$p_{10,3}$	$p_{10,4}$	$p_{10,5}$	$p_{10,6}$	$p_{10,7}$	$p_{10,8}$	$p_{10,9}$	$p_{10,10}$

Note: R = Condition Rating

S = State

$p_{i,j}$ = Transition Probability from State i to State j

2.5 Transition Probability Matrix

The inspection of bridges includes ratings of individual components such as deck, superstructure and substructure as well as of the overall bridge condition. Unless rehabilitation or repair is applied, bridge structures would be gradually deteriorating so that the bridge condition ratings are either unchanged or changed to a lower number in one year period. Therefore, the probability $p_{i,j}$ is null for $i > j$, where i and j represent the states in the Markov chain.

Since the rate of deterioration of bridge condition is different at different bridge ages, the transition process of bridge conditions is not homogeneous with respect to bridge age. However, a Markov process requires a presumption of homogeneity [Bhat 1972]. Therefore, if only one transition matrix were used throughout a bridge's life span, the inaccuracy of condition estimation would occur as a result of nonhomogeneity of the condition transition process. To avoid overestimating or underestimating the bridge condition, an approach, named a zoning technique, was used to obtain the transition matrix. This approach was used for the development of pavement performance curves in a previous study [Butt et al. 1987].

One year transition period was used in developing performance curves. In other words, $p_{i,j}$ was the transition probability from state i to state j in one year period. Bridge age was divided into groups and within each age group the Markov chain was assumed to be homogeneous. A six year group was found appropriate for the data base as well as for solving equations of unknown probabilities. A separate transition matrix was developed for each group.

To make the computations simple, an assumption was made that the bridge condition rating would not drop by more than one state in a single year. Thus, the bridge condition would either stay in its current state or transit to the next lower state in one year. The transition matrix of condition ratings has, therefore, the form:

$$P = \begin{vmatrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \tag{2.5}$$

where q(i)=1-p(i). p(i) is corresponded to $p_{i,i}$ and q(i) to $p_{i,i+1}$ in Table 2.5. Therefore, p(1) is the transition probability from rating 9 (state 1) to rating 9, and q(1), from rating 9 to rating 8, and so on.

It should be noted that the lowest rating number before a bridge is repaired or replaced is 3. Consequently, the corresponding transition probability p(7) equals to 1.

To estimate the transition matrix probabilities, for each age group the following non-linear programming objective function was formulated:

$$\min \sum_{t=1}^{N} \left| Y(t) - E(t,P) \right| \tag{2.6}$$

subject to: $0 \le p(i) \le 1, \quad i = 1, 2, \ldots, I$

where,

N = 6, the number of years in one age group,

I = 6, the number of unknown probabilities,

$P = [p(1), p(2), \ldots, p(I)]$, a vector of length I,

$Y(t)$ = the average of condition ratings at time t, estimated by regression function,

$E(t,P)$ = Estimated value of condition rating by Markov chain at time t.

The objective function was to minimize the absolute distance between the actual bridge condition rating at a certain age and the predicted bridge condition for the corresponding age generated by the Markov chain with the probabilities obtained by the non-linear programming. The solution to this function was obtained by utilizing a special FORTRAN program subroutine for solving non-linear programming. The subroutine uses the Quasi-Newton method [Luenberger 1984] and is available on the Engineering Computer Network system at Purdue University. The values of the corresponding regression function were taken as the average condition ratings to solve the non-linear programming.

The maximum rating of bridge condition is 9 and it represents a near-perfect condition of a bridge component. It is almost always true that a new bridge has condition rating 9 for all of its deck, superstructure and substructure. In other words, a bridge at age 0 has condition rating 9 for its components with unit probability. Thus, the initial state vector $Q_{(0)}$ for deck, superstructure or substructure of a new bridge is always [1, 0, 0,...,0], where the numbers are the probabilities of having condition rating of 9, 8, 7, ..., and 0 at age 0, respectively. That is, the initial vector of the first group for developing the bridge performance curve is known. Group 2 takes the last state

vector of group 1 as its starting state vector. Similarly, group n takes the last state vector of group n-1 as its starting state vector. The rest of the work to obtain the overall bridge performance curve or performance curve for bridge components is nothing but to conduct the following matrix multiplications:

$$
\begin{aligned}
Q_{(1)} &= Q_{(0)} * P \\
Q_{(2)} &= Q_{(0)} * P^2 \\
Q_{(3)} &= Q_{(0)} * P^3 \\
&\;\;\vdots \\
Q_{(t-1)} &= Q_{(0)} * P^{t-1} \\
Q_{(t)} &= Q_{(0)} * P^t
\end{aligned}
\tag{2.7}
$$

where, $Q_{(t)}$ represents the condition state vector at age t.

Let R be a vector of condition ratings, R=[9 8 7 6 5 4 3], and R' be the transform of R, i.e.,

$$
R' = \begin{vmatrix} 9 \\ 8 \\ 7 \\ 6 \\ 5 \\ 4 \\ 3 \end{vmatrix}
$$

then the estimated condition rating at age t by Markov chain is,

$$
E(t,P) = Q_{(t)} * R' \tag{2.8}
$$

For example, the performance function for substructures of steel bridges

on interstate highways gives the following values of predicted condition ratings for the first six years:

$$
\begin{aligned}
Y(1) &= 8.67;\\
Y(2) &= 8.37;\\
Y(3) &= 8.10;\\
Y(4) &= 7.86;\\
Y(5) &= 7.64;\\
Y(6) &= 7.44.
\end{aligned}
$$

The predictions of the condition ratings by the Markov chain method can be expressed by the following functions:

$$
\begin{aligned}
E(1,P) &= Q_{(0)}*P*R';\\
E(2,P) &= Q_{(0)}*P^2*R';\\
E(3,P) &= Q_{(0)}*P^3*R';\\
E(4,P) &= Q_{(0)}*P^4*R';\\
E(5,P) &= Q_{(0)}*P^5*R';\\
E(6,P) &= Q_{(0)}*P^6*R'.
\end{aligned}
$$

Since E(t,P)'s are functions of p1, p2, p3, p4, p5, and p6, Equation 2.6 can be solved to find the values of these probabilities. The transition matrices or transition probabilities for the Markov chain prediction model are found in Tables 2.6 through 2.17. Because the non-linear program may yield solutions with probability values of 0.0 or 1.0, some judgment and additional constraints were used as needed in the process of solving Equation 2.6 in order to obtain realistic solutions. Since the transition probabilities were obtained on the basis of performance functions, the values of transition probabilities are closely related to the shape of performance curves. For instance, examining Table 2.6 and Figure 2.1, it can be found that the values of p(3) and p(4) are greater between age 7 and age 30, where the performance curve is flat, than those between age 0 and age 6 and between age 31 and age 36, where the performance curve drops fast. Figure 2.5 shows a comparison of predictions of substructure conditions of concrete bridges on interstate highways by regression method and

Table 2.6 Transition Probabilities for Deck Condition (Concrete, Interstate)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.687	0.714	0.801	0.344	0.213	0.203
7 - 12	0.680	0.850	0.950	0.900	0.700	0.600
13 - 18	0.680	0.900	0.980	0.950	0.700	0.600
19 - 24	0.636	0.850	0.980	0.980	0.850	0.800
25 - 30	0.560	0.615	0.980	0.980	0.500	0.400
31 - 36	0.500	0.568	0.830	0.800	0.300	0.200

Table 2.7 Transition Probabilities for Superstructure Condition
(Concrete, Interstate)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.729	0.722	0.687	0.319	0.205	0.201
7 - 12	0.720	0.950	0.950	0.940	0.800	0.700
13 - 18	0.720	0.940	0.950	0.940	0.800	0.604
19 - 24	0.600	0.970	0.970	0.970	0.750	0.600
25 - 30	0.500	0.970	0.970	0.970	0.615	0.397
31 - 36	0.500	0.551	0.970	0.385	0.320	0.291

Table 2.8 Transition Probabilities for Substructure Condition (Concrete, Interstate)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.700	0.734	0.840	0.351	0.215	0.204
7 - 12	0.700	0.950	0.950	0.792	0.421	0.302
13 - 18	0.680	0.950	0.950	0.900	0.700	0.600
19 - 24	0.593	0.960	0.960	0.960	0.900	0.750
25 - 30	0.500	0.950	0.950	0.930	0.800	0.700
31 - 36	0.430	0.673	0.950	0.556	0.539	0.415

Table 2.10 Transition Probabilities for Superstructure Condition (Steel, Interstate)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.478	0.768	0.848	0.529	0.424	0.304
7 - 12	0.446	0.779	0.940	0.653	0.506	0.335
13 - 18	0.420	0.950	0.980	0.950	0.950	0.830
19 - 24	0.400	0.950	0.980	0.970	0.940	0.830
25 - 30	0.370	0.789	0.960	0.890	0.400	0.400
31 - 36	0.330	0.577	0.500	0.340	0.300	0.300

Table 2.11 Transition Probabilities for Substructure Condition (Steel, Interstate)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.585	0.657	0.900	0.561	0.437	0.312
7 - 12	0.580	0.792	0.950	0.800	0.644	0.361
13 - 18	0.580	0.970	0.980	0.970	0.950	0.890
19 - 24	0.570	0.950	0.980	0.980	0.970	0.930
25 - 30	0.520	0.648	0.950	0.800	0.624	0.375
31 - 36	0.400	0.400	0.400	0.400	0.300	0.200

Table 2.12 Transition Probabilities for Deck Condition
(Concrete, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.700	0.780	0.874	0.600	0.500	0.400
7 - 12	0.690	0.770	0.870	0.720	0.610	0.540
13 - 18	0.690	0.780	0.950	0.850	0.760	0.660
19 - 24	0.616	0.720	0.980	0.970	0.930	0.850
25 - 30	0.560	0.700	0.980	0.980	0.950	0.940
31 - 36	0.520	0.680	0.980	0.980	0.970	0.960
37 - 42	0.480	0.620	0.980	0.980	0.970	0.960
43 - 48	0.460	0.600	0.980	0.980	0.930	0.900
49 - 54	0.440	0.570	0.970	0.960	0.900	0.880
55 - 60	0.400	0.500	0.800	0.820	0.750	0.600

Table 2.13 Transition Probabilities for Superstructure Condition (Concrete, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.700	0.780	0.940	0.910	0.581	0.436
7 - 12	0.600	0.640	0.940	0.910	0.580	0.430
13 - 18	0.580	0.600	0.940	0.910	0.580	0.430
19 - 24	0.560	0.600	0.960	0.950	0.750	0.589
25 - 30	0.550	0.580	0.970	0.960	0.800	0.640
31 - 36	0.540	0.570	0.980	0.970	0.870	0.780
37 - 42	0.530	0.560	0.980	0.980	0.950	0.880
43 - 48	0.520	0.540	0.980	0.980	0.950	0.900
49 - 54	0.500	0.520	0.940	0.910	0.862	0.800
55 - 60	0.450	0.490	0.850	0.800	0.700	0.650

Table 2.14 Transition Probabilities for Substructure Condition (Concrete, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.704	0.741	0.850	0.800	0.700	0.650
7 - 12	0.600	0.710	0.940	0.800	0.700	0.650
13 - 18	0.550	0.640	0.940	0.936	0.700	0.650
19 - 24	0.550	0.640	0.950	0.950	0.800	0.750
25 - 30	0.540	0.610	0.970	0.970	0.910	0.860
31 - 36	0.530	0.600	0.985	0.985	0.970	0.970
37 - 42	0.520	0.580	0.985	0.985	0.970	0.970
43 - 48	0.500	0.550	0.985	0.985	0.970	0.970
49 - 54	0.480	0.530	0.944	0.950	0.840	0.840
55 - 60	0.450	0.500	0.800	0.800	0.700	0.600

Table 2.15 Transition Probabilities for Deck Condition
(Steel, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.646	0.676	0.950	0.868	0.752	0.704
7 - 12	0.600	0.630	0.850	0.800	0.700	0.650
13 - 18	0.600	0.630	0.900	0.900	0.800	0.700
19 - 24	0.580	0.620	0.950	0.950	0.900	0.800
25 - 30	0.560	0.590	0.960	0.960	0.930	0.860
31 - 36	0.540	0.570	0.980	0.980	0.950	0.890
37 - 42	0.530	0.550	0.980	0.980	0.960	0.910
43 - 48	0.530	0.550	0.900	0.900	0.850	0.830
49 - 54	0.510	0.530	0.800	0.800	0.700	0.650
55 - 60	0.500	0.510	0.750	0.750	0.550	0.500

Table 2.16 Transition Probabilities for Superstructure Condition (Steel, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.654	0.710	0.900	0.900	0.750	0.700
7 - 12	0.600	0.680	0.850	0.850	0.750	0.700
13 - 18	0.600	0.680	0.920	0.920	0.800	0.750
19 - 24	0.600	0.680	0.950	0.950	0.870	0.850
25 - 30	0.580	0.660	0.980	0.980	0.940	0.900
31 - 36	0.580	0.660	0.980	0.980	0.940	0.900
37 - 42	0.560	0.640	0.980	0.980	0.950	0.910
43 - 48	0.560	0.640	0.950	0.950	0.900	0.850
49 - 54	0.540	0.620	0.800	0.800	0.780	0.760
55 - 60	0.520	0.600	0.650	0.650	0.600	0.560

Table 2.17 Transition Probabilities for Substructure Condition
(Steel, Other Highways)

Age	p(1)	p(2)	p(3)	p(4)	p(5)	p(6)
0 - 6	0.670	0.700	0.900	0.900	0.786	0.685
7 - 12	0.650	0.700	0.848	0.859	0.750	0.699
13 - 18	0.650	0.700	0.920	0.920	0.900	0.900
19 - 24	0.650	0.700	0.950	0.950	0.920	0.920
25 - 30	0.620	0.647	0.980	0.980	0.950	0.950
31 - 36	0.620	0.640	0.980	0.980	0.950	0.950
37 - 42	0.600	0.640	0.980	0.980	0.970	0.960
43 - 48	0.600	0.620	0.980	0.980	0.970	0.960
49 - 54	0.560	0.580	0.850	0.860	0.600	0.560
55 - 60	0.560	0.600	0.650	0.660	0.450	0.400

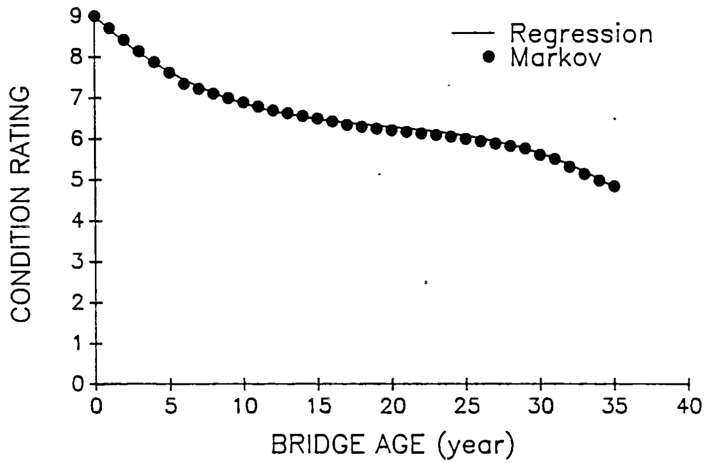

Figure 2.5 Comparison of Markov and Regression Condition Predictions

by Markov chain method. It can be seen that the two predictions were very close.

Even though the bridge performance curves have been developed by using regression method, it is still necessary to use the Markov chain model for the prediction of individual bridge conditions. As a matter of fact, both the performance curve and the Markov chain model play important but distinct roles in a bridge management system. A performance curve can be used to estimate the extent of condition improvement as a measure of effectiveness in selecting rehabilitation and repair strategies, as discussed in Chapter 3. However, when the condition prediction is concerned, the Markov chain model provides more reasonable estimation of bridge conditions, which is explained through examples in the following section.

2.6 Applications of the Markov Chain Model

Once the transition matrix is obtained, the prediction of the future condition by Markov chain becomes a matter of multiplication of matrices. Let us take the deck performance curve of concrete bridges on other state highways as an example. As mentioned earlier, the initial state vector of the first group for deck, superstructure or substructure of a new bridge is always [1, 0, 0,..., 0]. Therefore, the major problem is to obtain the transition matrix for bridge decks.

The values of Y(t) obtained from the performance function were used to solve the non-linear programming in Equation 2.6. This solution provided transition probabilities for different bridge age groups. For example, Table 2.12 shows the transition probabilities for deck condition of concrete bridges on non-interstate state highways for ten age groups. For illustration,

p(1)=0.700 for group 1 indicates that the probability of deck condition of bridges in group 1 (age 6 years or less) transiting from state 1 (condition rating 9) to state 1 (remaining in state 1) in one year period is 0.700, and the probability of transiting from state 1 to state 2 (condition rating 8) is q(1)=0.300. Similarly, p(2)=0.780 for group 1 indicates that the probability of deck condition of bridges in age group 1 transiting from state 2 to state 2 (remaining in state 2) in one year period is 0.780, and the probability of transiting from state 2 to state 3 (condition rating 7) is q(2)=0.222.

An example set of computations is given in the following. Using Equation 2.5 and information from Table 2.12, the transition matrix for age group 1 was obtained:

$$P = \begin{vmatrix} 0.700 & 0.300 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.780 & 0.220 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.874 & 0.126 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.600 & 0.400 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.500 & 0.500 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.400 & 0.600 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{vmatrix} \qquad (2.9)$$

The initial state vector of age group 1 was $Q_{(0)} = [1, 0, \ldots, 0]$. Therefore, the state vector and condition rating of age group 1 for year t can be obtained by Equations 2.7 and 2.8. For example, the state vectors and condition ratings for year 0 through year 6 are given below:

R = [9 8 7 6 5 4 3]

$Q_{(0)} = [1\ 0\ 0\ 0\ 0\ 0\ 0]$

$E(0,P) = Q_{(0)} * R' = 9.0$

$Q_{(1)} = Q_{(0)} * P = [0.70\ 0.30\ 0.00\ 0.00\ 0.00\ 0.00\ 0.00]$

$E(1,P) = Q_{(1)} * R' = 8.70$

$Q_{(2)} = Q_{(0)} * P^2 = [0.49\ 0.44\ 0.07\ 0.00\ 0.00\ 0.00\ 0.00]$

$E(2,P) = Q_{(2)} * R' = 8.42$

$Q_{(3)} = Q_{(0)} * P^3 = [0.34\ 0.49\ 0.16\ 0.01\ 0.00\ 0.00\ 0.00]$

$E(3,P) = Q_{(3)} * R' = 8.17$

$Q_{(4)} = Q_{(0)} * P^4 = [0.24\ 0.49\ 0.24\ 0.03\ 0.00\ 0.00\ 0.00]$

$E(4,P) = Q_{(4)} * R' = 7.94$

$Q_{(5)} = Q_{(0)} * P^5 = [0.17\ 0.45\ 0.32\ 0.05\ 0.01\ 0.00\ 0.00]$

$E(5,P) = Q_{(5)} * R' = 7.72$

$Q_{(6)} = Q_{(0)} * P^6 = [0.12\ 0.40\ 0.38\ 0.07\ 0.02\ 0.01\ 0.00]$

$E(6,P) = Q_{(6)} * R' = 7.50$

Then, $Q_{(6)}$ obtained above for group 1 was taken as the initial state vector of group 2 and the corresponding transition matrix of group 2 was used to continue the procedure. By this procedure, the bridge condition at any time t can be predicted in terms of initial state vector, $Q_{(0)}$, and transition matrix, P.

Since a performance curve of bridges represents the average or mean condition rating at any given bridge age, the above example indicates that both Markov chain method and regression method can be used to predict the average condition ratings of bridges. However, the following example shows that the Markov chain method has great advantages over the regression method in predicting conditions of individual bridges. Figure 2.6 presents the performance curve of concrete bridge decks, a bridge is presently 10 years old with deck condition rating 6, which is denoted by r_{10}. It is desired to predict the deck condition rating at bridge age 15, i.e., to predict the deck condition rating in 5 years. Using the Markov chain model, the deck condition is predicted as $r_{15}^{M} = 4.79$.

The regression model gives a prediction of $r_{15}^{R} = 6.21$, which is even greater than the current rating value and therefore is apparently inaccurate. As can be seen, the regression model is appropriate only in estimating the average condition rating of a group of bridges. However, the Markov chain model is useful in estimating both the average condition rating of bridges and the condition rating of a particular bridge.

Regression-based performance curves have been used for condition prediction in many pavement management studies. To predict pavement condition, a performance curve is modified by shifting the curve up or down to match the current condition rating. Then the shifted curve is used to determine the future condition rating. However, this approach is not correct, because it assumes that the deterioration rate of condition is independent of current condition rating. For instance, using a shifted performance curve, a 10 year old bridge with condition rating of 6 and a 10 year old bridge with condition rating of 7 would experience the same amount of drop in condition rating in one year period.

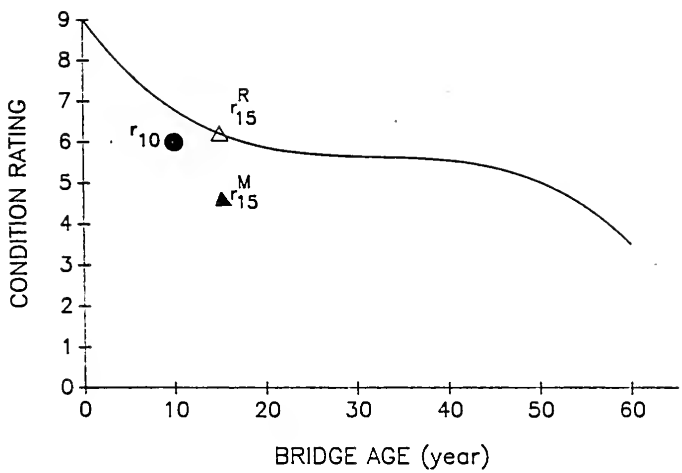

Figure 2.6 Example of Condition Prediction

However, according to Table 2.12, the probabilities of remaining in the same condition rating in one year period for these two bridges are 0.72 and 0.87 (p(4) and p(3) for bridge ages between 7 and 12), respectively. That is, the bridge with lower condition rating (6) is expected to deteriorate more quickly. This indicates that in the Markov chain model the deterioration rate of a bridge condition is determined not only by the bridge age, but also by the current condition of the bridge. Therefore, the Markov chain model provides a more realistic and comprehensive picture of bridge condition changes.

2.7 Chapter Conclusions

It is essential to estimate accurately the future conditions of bridges for an effective bridge management system. Markov chain theory is a powerful and convenient tool for estimating future bridge performance. The results obtained by Markov chain model are particularly useful if a dynamic programming is used for optimization in a bridge management system, since the transition probabilities are the basic parameters to determine before one can solve a dynamic programming [Ross 1970]. Furthermore, performance curves give bridge managers a quantitative view of bridge conditions that are useful in selecting rehabilitation strategies.

A Markov chain is completely specified when its transition matrix P and the initial state vector $Q_{(0)}$ are known. Usually, the initial condition is known in a bridge management system. So the main task before using the Markov chain is to develop the transition probability matrix.

Since the transition matrices for all the subgroups of bridges in Indiana were developed in this study, the Markov model has been completely developed and

can be used to predict condition ratings of the state highway bridges in Indiana. Also, the performance functions obtained by this study provide a measure of effectiveness of bridge rehabilitation for the optimization model presented in the next chapter.

CHAPTER 3: DYNAMIC OPTIMIZATION MODEL OF PROJECT SELECTION

3.1 Introduction

Several states, including Pennsylvania, North Carolina, Virginia, Nebraska and Kansas, have developed relatively comprehensive bridge management systems [FHWA 1987b]. However, all of these systems are based on priority ranking techniques to select bridge improvement projects, which usually do not guarantee optimal solutions. Mathematical techniques of optimization have not yet been effectively used in bridge management systems.

Ranking techniques sort projects in priority order through evaluation of several factors for each project in the system. The projects are usually selected from the top of the priority list until the available budget is used up. This approach to selecting projects is virtually based on the rule of "choosing the project with the worst conditions". Although this rule is considered rational by many decision-makers and is widely adopted in the project selection practice, it does not maximize benefit or minimize negative effects of a system. On the other hand, optimization techniques manipulate the tradeoffs between the objective and constraints systematically or mathematically, so that an optimal solution to the problem among many possible solutions can be obtained. In managing a bridge system, optimization techniques can be applied to produce optimal strategies in project selection by maximizing the system benefit subject to the constraints, such as available resources.

This chapter describes an optimization model developed for a comprehensive bridge management system for the Indiana Department of Transportation (INDOT).

The model applies dynamic programming and integer linear programming to select projects while the effectiveness or benefit of a bridge system is maximized subject to the constraints of available budgets over a given program period. The performance curves and the Markov chain prediction model of bridge conditions were incorporated in the optimization model. In one version of the model, the change of the area under a performance curve caused by a rehabilitation or replacement activity is used as a measure of effectiveness obtained by the activity. In another version, described in Chapter 4, the utility values used in the priority ranking model (Volume 5) are incorporated in the objective function. Markov chain transition probabilities of bridge conditions are used in the model to predict or update bridge conditions at each stage of the dynamic programming. The use of dynamic programming, in combination with integer linear programming and Markov chain, makes it possible to manage efficiently a system with hundreds of bridges. The application of dynamic programming assures that the results are not only optimal for a program period, but also for the subperiods.

3.2 Optimization

The concept of optimization is now applied as a principle underlying the analysis of many complex decision or allocation problems. Using optimization techniques, one approaches a complex decision problem, involving the solution of values for a number of interrelated variables, by focussing attention on a single objective designed to quantify performance and measure the quality of the decision [Luenberger 1965]. This one objective is maximized (or minimized depending on the formulation) subject to the constraints that may limit the selection of decision variable values. If a suitable single aspect of a problem

can be isolated and characterized by an objective, optimization may provide a suitable framework for analysis and produce the best solution to the problem from a set of alternatives.

In managing a bridge system, problems such as selecting projects to maximize system benefit with a limited budget are difficult because many related factors are involved. It is virtually impossible to fully represent all the complexities of variable interactions, constraints, and appropriate objectives when a statewide bridge system is considered. Thus, modelling a problem as well as formulating it quantitatively are actually processes of approximation. An optimal solution, then, should be regarded as the best solution corresponding to the specific formulation rather than the absolutely correct solution to the real system.

Many different optimization techniques, such as dynamic programming, linear programming, integer programming, and goal programming, have been applied to roadway management problems. The use of one technique instead of another depends on the nature of the given problem and various considerations of the model to be developed. Because the stochastic nature of bridge systems and the large number of variables involved in bridge project selection, the dynamic programming and integer linear programming in combination were chosen for the optimization model in the Indiana Bridge Management System.

3.3 Use of Integer Linear Programming

A zero-one integer linear programming [Gottfried and Weisman 1973], used in this model, is defined as:

maximize (or minimize)

$$Z = \sum_{j=1}^{p} c_j X_j \tag{3.1}$$

subject to

$$\sum_{j=1}^{p} a_{ij} X_j \ (\leq, =, \geq) \ b_i \qquad i = 1, 2, \ldots, m \tag{3.2}$$

$$X_j = (0, 1) \qquad j = 1, 2, \ldots, p \tag{3.3}$$

where c_j, a_{ij}, and b_i are known constants for all i and j, and X_j are variables with values of 0 or 1 for all j.

This technique is a well-defined procedure and can be used to maximize benefit or minimize cost subject to a number of constraints. In developing the bridge management system, three major rehabilitation activities, deck reconstruction, deck replacement and bridge replacement, were considered. Each activity of a bridge was defined as a zero-one decision variable. When the value of one of the decision variables is one, the corresponding activity is selected; otherwise, routine maintenance is assumed for the bridge. The objective function of the integer linear programming was to maximize the system effectiveness in each year.

3.4 Introduction to Dynamic Programming

Dynamic programming is a particular approach to optimization [Bellman 1957]. It is not a specific algorithm in the sense that Simplex algorithm is a well-defined set of rules for solving a linear programming problem or in the sense that branch-and-bound is a well-defined procedure for finding the optimal solution of an integer programming. Dynamic programming is a way of looking at a problem which may contain a large number of interrelated decision variables so

that the problem is regarded as if it consisted of a sequence of problems, each of which required the determination of only one (or a few) variables [Cooper and Cooper 1981].

Dynamic programming approach substitutes n single variable problems for solving one n variable problem, so that it usually requires much less computational effort. The principle that makes the transformation of a n-variable problem to n single variable problems possible is known as the principle of optimality, which is stated as:

> An optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with respect to the state which results from the initial decision [Cooper and Cooper 1981].

A simpler expression of this principle consists of the following statement: "every optimal policy consists only of optimal subpolicies" [Cooper and Cooper 1981].

An important advantage of dynamic programming is that it determines absolute (global) maxima or minima rather than relative (local) optima. Also, dynamic programming can easily handle integrality and non-negativity of decision variables. Furthermore, the principle of optimality assures that dynamic programming results in not only the optimal solution of a problem, but also the optimal solutions of subproblems. For example, for a 10 year program period, dynamic programming gives the optimal project selections for the entire 10 year period as well as the optimal project selections for any period less than 10

years. These optimal solutions of the subperiods are often of interest to bridge managers.

The key elements of a dynamic programming are stages, states, decision and return [Cooper and Cooper 1981]. A bridge system can be considered to progress through a series of consecutive stages, each year is viewed as a stage. At each stage, the system is described by states, such as bridge condition and available budget. Decisions (project selections) are made at each stage by optimizing the returns (system benefit). The bridge conditions are predicted and updated by Markov chain technique and the system undergoes the next stage. A major limitation of dynamic programming is that if there are too many state variables and decision variables, then we have computational problems relating to the storage of information as well as the time it takes to perform the computation.

3.5 Optimization Model

The proposed optimization model for the Indiana Bridge Management System requires that it handle about 1,000 bridges with about 3,000 decision variables, if only 3 improvement alternatives are considered (deck reconstruction, deck replacement and bridge replacement). Furthermore, each bridge has a number of associated factors such as condition rating, traffic safety index, community impact index, and so on. Because of the size of the problem, it was not possible to use only dynamic programming to optimize such a large system. Therefore, integer linear programming was used in combination with dynamic programming to optimize the project selections on a statewide basis.

The dynamic programming divides the federal and state budgets of each year into several possible spending portions and the integer linear programming

selects projects by maximizing yearly system effectiveness subject to different given budgets. The dynamic programming chooses the optimal spending policy, which maximizes the system effectiveness over a program period, by comparing the values of effectiveness of these given budgets resulted by the integer linear programming for each year. For example, suppose the program period T equals to 2 years, and the possible spendings for year 1 are 50, 60, 70, 80, 90 and 100 millions, and the possible spendings for year 2 are 150, 140, 130, 120, 110 and 100 millions, respectively. Any combination of spendings for the individual years can be considered. The task of the dynamic programming is to determine the optimal policy among possible combinations of spendings, i.e., (50, 150), (60, 140), (70, 130), (80, 120), (90, 110) and (100, 100), and to obtain the corresponding optimal project selections. Similarly, if T is larger than 2, say 10, the model can determine the optimal policy from year 1 to year 10 and give the corresponding project selections.

In terms of dynamic programming, each year of the program period is a stage. The federal and state budgets are state variables. Each activity of a bridge is a decision variable of the dynamic programming as well as of the integer linear programming. The effectiveness of the entire system is used as the return of the dynamic system.

At each stage, decision must be made as to the optimal solution from stage 1 to the current stage. When a decision is made, a return (or reward) is obtained and the system undergoes a transformation to the next stage. The bridge conditions are updated for the next stage by the Markov transition probabilities obtained by the performance model described in Chapter 2. Figure 3.1 is a flow chart of the optimization model which illustrates the optimization process. For

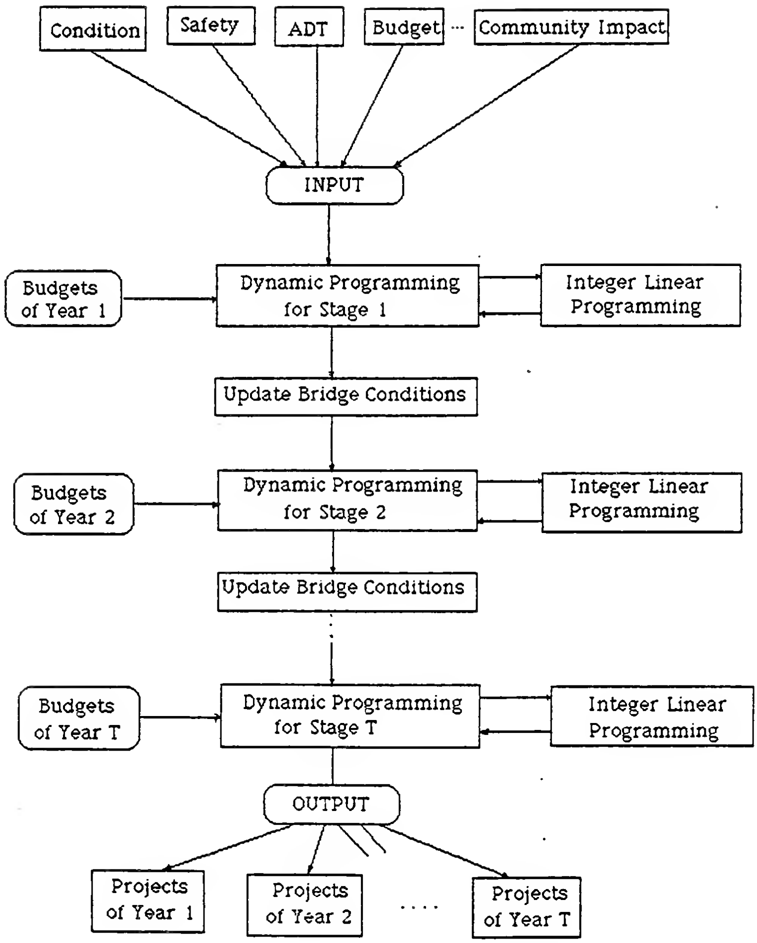

Figure 3.1 Flow Chart of the Optimization Model

a given program period, the objective of the model is to maximize the effectiveness of the entire system. The definition of system effectiveness and some other assumptions are discussed as follows.

3.5.1 Assumptions and Definitions

The purpose of the dynamic optimization model is to select bridge projects that would provide the maximum systemwide benefit within a given budget. Different bridge deficiency problems call for different treatments. To develop the optimization model, the bridge activities must be clearly identified and defined. In Indiana, rehabilitation activities mainly include deck reconstruction and deck replacement. Deck reconstruction work includes shallow and/or full-depth patching of deteriorated deck spots and an overlay of the deck after scarifying the wearing surface. Along with this reconstruction, curbs, railing, and expansion joints are replaced in most cases. Other related works include guardrails, approach slab reconstruction, approach shoulder reconstruction, and small amounts of substructure repairs. The deck replacement alternative is a more extensive rehabilitation work than deck reconstruction. Deck replacement consists of a replacement of the entire deck, including rehabilitation of parts of the superstructure and the top portion of the substructure. The replacement of the entire bridge is considered when reconstruction and rehabilitation cannot adequately correct the existing deficiencies. Thus, bridge rehabilitation and replacement activities were grouped into three options:

1. Deck reconstruction;
2. Deck replacement; and
3. Bridge replacement.

A detailed cost analysis for these three activities was conducted and discussed in Volume 4 of this report. The results of this anlaysis can be used to estimate costs of the activities.

When a rehabilitation activity is applied on a bridge, the condition rating of various bridge components increases depending on the type of improvement. As shown in Figure 3.2, a particular rehabilitation activity causes a jump in the deck condition rating. As the bridge age increases, the condition rating gradually decreases from the new condition rating. The area between the deterioration curves of bridge i with and without rehabilitation a, $A_i(a)$, represents an improvement in terms of condition rating and service life of the bridge.

From a performance curve, one can see that the deterioration rate of bridge condition is different at different ages. Figure 3.3 shows a tangent line on a performance curve, the absolute value of $tg\alpha$, d_i, is the deterioration rate at the corresponding time. It is evident that when the value of deterioration rate is small, a rehabilitation may be delayed for a period of time. Otherwise, an immediate repair should be applied because a rapid deterioration is expected. It should be noted that since a bridge with a condition rating greater than 6 does not need rehabilitation, the tangent values on the curve section between condition rating of 6 and 9 are not considered in selecting bridge activities in the optimization model.

In order to develop an optimization program, the first task is to define the objective function of the program. As discussed above, the area $A_i(a)$ shown in Figure 3.2 represents the condition improvement that can be expected from

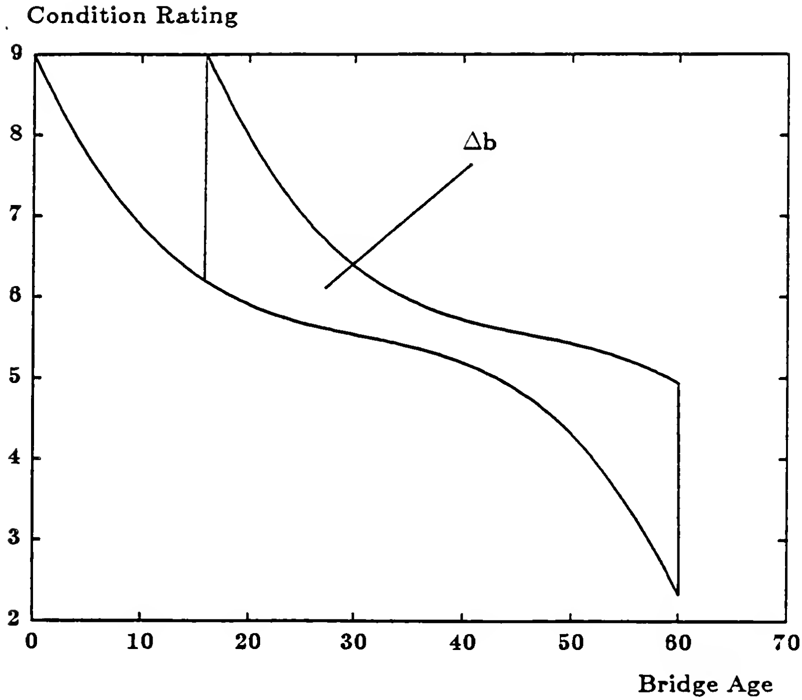

Figure 3.2 Area of Performance Curve Obtained by Rehabilitation

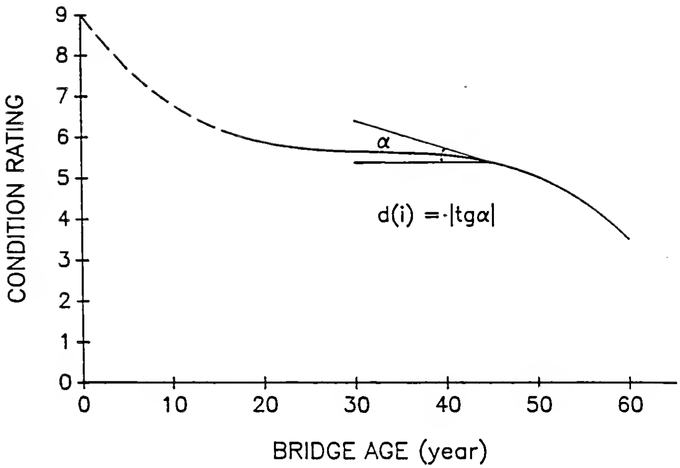

Figure 3.3 Slope of Performance Curve

undertaking a rehabilitation or replacement activity. In addition, other factors, such as average daily traffic (ADT), traffic safety condition, and community impact of a bridge, should also be considered in the optimization program.

There are several ways the effectiveness of a bridge activity can be defined. Because ADT_i is the number of vehicles served by bridge i, the multiplication of ADT_i and $A_i(a)$, $ADT_i * A_i(a)$, can be interpreted as the measure of the improvement that can be experienced by the users or vehicles on bridge i. Traffic safety condition and community impact of a bridge are two other factors affecting decisions on bridge rehabilitation or replacement activities in addition to structural condition. "Bridge safety index" discussed in Volume 3 of this report and bridge detour length were used as variables reflecting bridge traffic safety and community impact, respectively. To determine the effects of these factors, a group of INDOT bridge engineers were interviewed and utility curves were developed, as shown in Figures 3.4 and 3.5. These corves convert the factor effects into dimensionless coefficients. These coefficients were then used to modify the effectiveness of individual bridge projects depending on site specific impacts.

The effectiveness of a bridge improvement activity was defined as follows:

$$E_i = ADT_i * \Delta A_i(a) * (1 + Csafe_i) * (1 + Cimpc_i) \qquad (3.4)$$

where:

E_i = effectiveness gained by bridge i if activity a is chosen;

a = improvement activity:

a = 1, deck reconstruction;

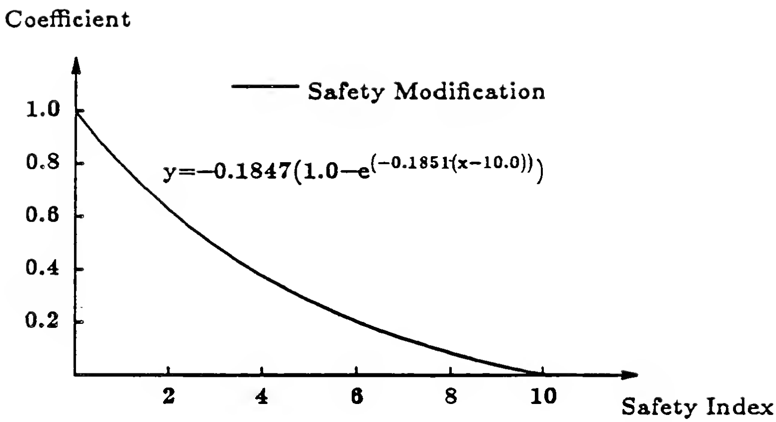

Figure 3.4 Coefficient of Traffic Safety Index

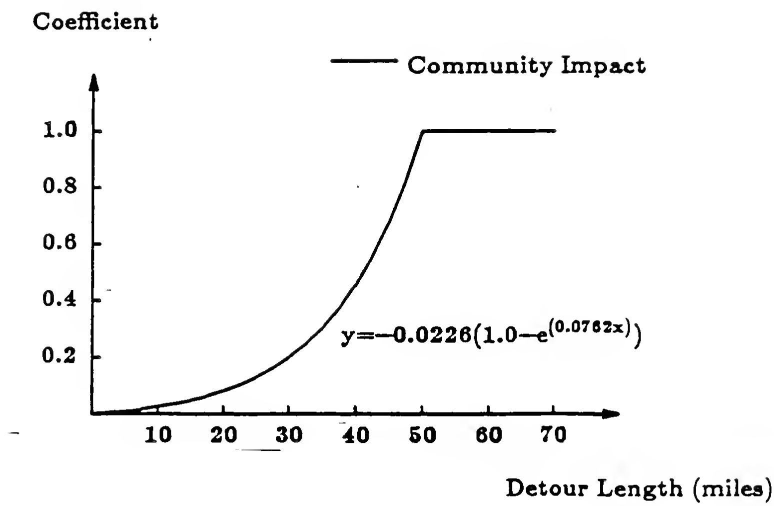

Figure 3.5 Coefficient of Community Impact

a = 2, deck replacement;

a = 3, bridge replacement;

ADT_i = average daily traffic on bridge i;

$\Delta A_i(a)$ = $f_1\Delta b_{i1} + f_2\Delta b_{i2} + f_3\Delta b_{i3}$, representing average value of areas under performance curves of components of bridge i obtained by activity a, where f_j's are the frequencies of the corresponding component being repaired in activity a, Δb_{ij}'s are the areas of the component gained by activity a, with j=1, 2 and 3 corresponding to deck, superstructure and substructure, respectively. Figure 3.2 shows an example of Δb_{i1}, i.e., the area obtained under the performance curve of deck condition.

$Csafe_i$ = transformed coefficient of traffic safety index (primarily based on bridge geometrics) of bridge i, as shown in Figure 3.4; the safety index ranges from 1 to 10 with 10 being the index of no potential traffic safety problem.

$Cimpc_i$ = transformed coefficient of community impact of bridge i in terms of detour length, as shown in Figure 3.5.

3.5.2 Formulation

Considering that budgets can be carried over from year to year, the mathematical model for maximizing the overall effectiveness of various activities over a program period T was formulated:

$$\max \sum_{t=1}^{T} [\sum_{i} \sum_{a} X_{i,t}(a) * E_i * d_i(t)] \tag{3.5}$$

Subject to the following constraints:

(*a*) available federal budget,

$$\sum_{t=1}^{T} [\sum_{i} \sum_{a} X_{i,t}(a) * c_i(a) * F_i] \leq C_{BF} \tag{3.6}$$

(b) available state budget,

$$\sum_{t=1}^{T} [\sum_{i} \sum_{a} X_{i,t}(a) * c_i(a) * (1 - F_i)] \leq C_{BS} \tag{3.7}$$

(c) one activity can not be undertaken more than once on one bridge in T years,

$$\sum_{t=1}^{T} X_{i,t}(a) \leq 1 \tag{3.8}$$

Constraints (f) to (j) correspond to the integer linear programming problem:

(f) maximize system effectiveness of year t,

$$\max \sum_{i} \sum_{a} [X_{i,t}(a) * E_i * d_i(t)] \tag{3.9}$$

(g) spending constraint of year t for federal budget,

$$\sum_{i} \sum_{a} [X_{i,t}(a) * c_i(a) * F_i] \leq {}^{n}{}_{tF} \tag{3.10}$$

(h) spending constraint of year t for state budget,

$$\sum_{i} \sum_{a} [X_{i,t}(a) * c_i(a) * (1 - F_i)] \leq {}^{n}{}_{tS} \tag{3.11}$$

(i) no more than one activity can be chosen on one bridge in year t,

$$\sum_{a=1}^{3} X_{i,t}(a) \leq 1 \qquad (3.12)$$

(j) decision variable,

$$X_{i,t}(a) = 0 \text{ or } 1 \qquad (3.13)$$

The Markov chain model is incorporated into the optimization model to update bridge conditions:

If bridge i is not selected in year t,

$$R_{i,t+1} = R_{i,t}*p_i(R,t)+(R_{i,t}-1)*(1-p_i(R,t)) \qquad (3.14)$$

If bridge i is selected in year t for activity a, its condition will be improved,

$$R_{i,t+1} = R_{i,t} + \Delta R_i(a) \qquad (3.15)$$

where:

$X_{i,t}(a) = 1$, if bridge i is chosen for activity a;

$X_{i,t}(a) = 0$, otherwise;

$d_i(t) =$ the absolute tangent value on performance curve of bridge i at time t, as shown in Figure 3.3, $d_i(t)$ reflects the deterioration rate of bridge condition at time t;

$C_{BF} =$ total available federal budget for the program period;

$C_{BS} =$ total available state budget for the program period;

F_i = federal budget share of bridge i;

$1-F_i$ = state budget share of bridge i;

$c_i(a)$ = estimated cost of activity a on bridge i;

${}^{n}{}_{tF}$ = spending limit of federal budget in year t;

${}^{n}{}_{tS}$ = spending limit of state budget in year t;

$R_{i,t}$ = condition rating of bridge i in year t;

$p_i(R,t)$ = Markov condition transition probability of bridge i with condition rating R in year t;

$\Delta R_i(a)$ = condition rating gained by bridge i for activity a.

3.5.3 Solution Technique

Equations 3.5 through 3.13 constitute a dynamic programming which includes an integer linear program (Equations 3.9 to 3.13) as a part of the constraints. The objective of the model is to obtain optimal budget allocations and corresponding project selections over T years so that the system effectiveness can be maximized. Let us denote the number of spending combinations by N, the number of possible spendings of each year by s, and the program period by T, then N can be expressed by s and T, $N=s^{T-1}$. When T is large, the number of possible spending combinations becomes so large that the search for the optimal path of

spendings from year 1 to year T needs great effort and computation time.

Dynamic programming is an efficient technique to search for the optimal path among the combinations of spendings. Rather than examining all the paths, dynamic programming looks at only a small part of these paths. According to the principle of optimality, at each stage the programming finds the optimal subpath up to the current stage, and only this subpath is used to search for the optimal subpath up to the next stage. The paths that do not belong to the optimal subpath are abandoned as the search goes on, which makes the search efficient and saves a great deal of time.

The search for the optimal path can be easily performed by expressing the problem as recurrence relations [Cooper and Cooper 1981]. In doing so, Equations 3.5, 3.6 and 3.7 are rewritten as follows,

$$\max \sum_{t=1}^{T} \Phi_t(Y(t)) \tag{3.16}$$

subject to

$$\sum_{t=1}^{T} Y_F(t) \le C_{BF} \tag{3.17}$$

$$\sum_{t=1}^{T} Y_S(t) \le C_{BS} \tag{3.18}$$

where:

$$\Phi_t(Y(t)) = \sum_i \sum_a [X_{i,t}(a) * E_i * d_i(t)]$$

$$Y_F(t) = \sum_i \sum_a [X_{i,t}(a) * c_i(a) * F_i] \le C_{BF}$$

$$Y_S(t) = \sum_i \sum_a [X_{i,t}(a) * c_i(a) * (1 - F_i)] \le C_{BS}$$

$$Y(t) = Y_F(t) + Y_S(t)$$

We define state variable as:

$$\lambda_t = \lambda_{t+1} - Y(t+1) \tag{3.19}$$

We also define the optimal return function as:

$$g_1(\lambda_1) = \max \Phi_1(Y(1)), \quad 0 \le Y(1) \le \lambda_1 \tag{3.20}$$

$$g_2(\lambda_2) = \max[\Phi_2(Y(2)) + g_1(\lambda_2 - Y(2))], \quad 0 \le Y(2) \le \lambda_2 \tag{3.21}$$

$$g_t(\lambda_t) = \max[\Phi_t(Y(t)) + g_{t-1}(\lambda_t - Y(t))], \quad 0 \le Y(t) \le \lambda_t \tag{3.22}$$

By the recurrence relations of Equations 3.20, 3.21 and 3.22, the dynamic programming process starts at year 1, or stage 1, and $g_1(\lambda_1)$ can be obtained for all the possible spendings of year 1. Then the bridge conditions are updated by Equation 3.14 or Equation 3.15 according to the project selections corresponding to $g_1(\lambda_1)$, and $g_2(\lambda_2)$ can be solved based on the information of $g_1(\lambda_1)$ as well as the updated bridge conditions. This forward recursion is performed for every successive year of the program period until $g_T(\lambda_T)$ is obtained, and therefore the optimal spending policy and project selection from year 1 to year T are obtained.

The value of $\Phi_t(Y(t))$ can be obtained by solving the integer linear program (Equations 3.9 to 3.13). The value of the objective function (Equation 3.9) of the linear program equals $\Phi_t(Y(t))$ if η_{tF} and η_{tS} of Equations 3.10 and 3.11 are substituted by possible spending limitations of year t.

3.6 An Example Application

A computer program of the optimization model was coded in Fortran 77. XMP package [Marsten 1987] was used in the programming to solve the integer linear programming. The Branch-and-bound method [Gottfried and Weisman 1973] was applied to solve the integer linear programming, which is essentially a direct enumeration technique that excludes from considering a large number of possible integer combinations and, therefore, makes it possible to solve a problem with hundreds of decision variables. The input of the problem includes the following:

1. Condition ratings of bridge components;
2. Bridge age;
3. Bridge type;
4. Highway type;
5. Safety index;
6. Detour length;
7. Average daily traffic;
8. Available federal and state budgets;
9. Federal budget share for bridge projects by highway type, ranging from 0.0 to 1.0;
10. Recommended activity and timing by engineers;
11. Estimated rehabilitation cost;

12. Program period.

The output of the program is a list of selected bridges, activities and the corresponding costs for each year of the program period. The output of this model depends on the available budgets. As budget changes, the program gives different project selections so that the system effectiveness could be maximized by efficiently spending available budget in the program period.

To show the application of the model, an example problem is presented as follows. Fifty bridges are given, 25 of them are recommended for rehabilitation, and another 25 bridges are recommended for bridge replacement. Tables 3.1 and 3.2 give the general information that was provided by INDOT on 25 rehabilitation bridges and 25 replacement bridges, respectively. It includes a description of each bridge and the activities and timings recommended by bridge inspectors or engineers. A 5 year program period is used (i.e., T=5). Suppose the bridges being considered are eligible for a 90% federal budget share (F_i) on interstates and a 80% federal budget share on non-interstate highways. The program is run for different budget inputs and the outputs corresponding to these different budget scenarios are used to compare the project selection and the values of system effectiveness. Tables 3.3 and 3.4 present results obtained by available budgets equal to 100% and 40% of the needed budgets, respectively. Figure 3.6 shows the comparison of project selections and system benefits obtained with respect to the size of different available budgets. The results indicate that at a lower level of budget most of the projects are rehabilitations. This trend continues until about 60% level of budget. then, replacement projects increase at a higher rate, while rehabilitation projects start to decrease. The reason for a higher number of rehabilitation projects at lower budget levels is that

Table 3.1 Information Pertaining to 25 Rehabilitation Bridges

Bridge No.	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
Bridge Age	16	27	31	15	36	28	30	39	30	26	35	16	16
Bridge Type	S	S	S	S	S	S	S	S	C	S	C	S	S
Highway Type	N	N	N	N	N	I	I	N	N	I	I	N	N
ADT (x100)	63	70	104	42	10	103	108	285	26	130	999	61	61
Detour Length (miles)	6	1	2	1	7	0	0	9	4	1	2	1	1
Remaining Service Life	20	15	20	20	20	20	15	25	13	10	5	10	10
Deck Rating	6	8	4	6	5	8	4	5	6	5	6	6	5
Superstructure Rating	6	6	6	6	6	6	6	6	6	6	7	7	6
Substructure Rating	7	6	7	5	7	6	5	6	6	7	6	7	7
Structure Length (feet)	229	349	276	299	162	155	212	899	117	334	192	217	217
Clear Deck Width (feet)	44	30	54	40	28	30	39	48	94	39	52	40	40
Deck Width (feet)	47	36	64	43	30	36	45	57	100	45	64	43	.43
Estimated Cost ($1000)	235	276	387	281	107	121	210	1300	257	330	259	201	201

Bridge No.	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24	H25
Bridge Age	15	24	23	29	30	23	18	35	30	23	75	76
Bridge Type	S	C	C	S	C	S	C	S	C	C	S	S
Highway Type	I	I	I	N	N	N	N	N	I	N	N	N
ADT (x100)	327	165	65	26	36	31	52	369	119	8	235	12
Detour Length (miles)	1	1	1	1	4	4	3	9	3	8	3	12
Remaining Service Life	30	25	30	4	20	20	20	20	12	5	2	2
Deck Rating	6	7	6	3	5	6	6	6	6	4	6	4
Superstructure Rating	6	7	7	5	6	6	5	5	6	5	3	4
Substructure Rating	7	7	5	6	6	6	6	5	6	6	4	4
Structure Length (feet)	400	80	152	312	73	155	117	176	206	401	72	642
Clear Deck Width (feet)	52	40	43	71	39	33	44	61	32	31	44	15
Deck Width (feet)	55	43	52	77	41	37	47	64	37	34	63	16
Estimated Cost ($1000)	476	74	154	1090	66	124	119	247	164	296	98	1993

Table 3.2 Information Pertaining to 25 Replacement Bridges

Bridge No.	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
Bridge Age	52	56	49	49	65	14	47	80	69	58	42	74
Bridge Type	S	S	S	N	S	S	N	S	C	N	S	S
Highway Type	N	N	N	N	N	N	N	N	N	N	N	N
ADT (x100)	11	80	72	90	29	2	170	13	123	101	7	7
Detour Length (miles)	13	17	15	5	4	14	10	3	6	6	25	25
Remaining Service Life	1	5	5	8	1	1	2	1	5	5	9	5
Deck Rating	3	3	4	5	3	3	4	4	5	5	5	4
Superstructure Rating	3	3	4	5	3	3	4	3	5	5	4	4
Substructure Rating	2	4	6	5	2	2	3	3	6	5	4	4
Structure Length (feet)	158	603	1363	510	605	168	377	104	257	69	152	152
Clear Deck Width (feet)	24	24	24	22	19	15	28	16	46	46	14	14
Deck Width (feet)	27	25	27	27	20	16	35	18	65	50	17	17
Est. Cost ($1000)	500	5000	6229	2192	1210	2159	3409	545	1297	635	840	840

Bridge No.	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
Bridge Age	65	40	60	21	48	72	21	24	81	56	72	84	50
Bridge Type	S	S	C	C	C	S	C	C	S	S	C	S	C
Highway Type	N	N	N	N	N	N	N	N	N	N	N	N	N
ADT (x100)	96	9	17	40	212	34	8	6	5	35	68	4	
Detour (miles)	2	3	4	7	2	0	4	4	4	4	0	9	5
Remaining Life	8	8	2	8	2	1	1	1	1	1	2	4	1
Deck Rating	6	6	3	6	4	4	4	4	3	4	3	3	4
Super. Rating	3	7	3	6	3	4	3	4	3	4	3	3	2
Sub. Rating	5	4	6	6	3	3	3	3	3	5	2	5	3
Structure Length	202	98	73	169	68	94	24	66	204	214	65	815	36
Clear Deck Width	24	24	34	27	50	12	32	25	19	19	49	31	28
Deck Width	38	28	36	29	63	15	34	27	20	20	55	52	30
Est. Cost ($1000)	3154	295	193	385	1571	1029	388	288	965	1549	420	65	280

Table 3.3 Output of Optimization Program (100% Needed Budget)

YEAR	BRIDGE No.	ACTIVITY	FED. COST ($1000)	STATE COST ($1000)	TOTAL ($1000)
1	H7	DRC	189	21	8502
	H11	DRC	242	27	
	H14	DRC	428	48	
	H16	DRC	139	15	
	H21	DRC	198	49	
	H22	DRC	148	16	
	P7	BRP	2727	682	
	P9	BRP	1037	259	
	P10	BRP	508	127	
	P15	BRP	154	39	
	P18	BRP	823	206	
	P23	BRP	336	84	
2	H24	DRC	79	20	9124
	H25	DRC	1594	399	
	P1	BRP	400	100	
	P4	BRP	1754	438	
	P5	BRP	968	242	
	P8	BRP	536	109	
	P12	BRP	672	168	
	P14	BRP	236	59	
	P16	BRP	308	77	
	P21	BRP	772	193	
3	H2	DRC	221	55	8369
	H8	DRC	1040	260	
	H9	DRC	205	51	
	H18	DRC	53	13	
	H19	DRC	99	25	
	H20	DRC	95	24	
	P2	BRP	4000	1000	
	P11	BRP	672	168	
	P19	BRP	310	78	
4	H6	DRC	109	12	7749
	H10	DRC	297	33	
	H15	DRC	67	7	
	H23	DRC	237	59	
	P6	BRP	1727	432	
	P13	BRP	2522	631	
	P22	BRP	1239	310	
	P24	BRP	52	13	
5	H1	DRC	188	47	10969
	H3	DRC	310	77	
	H4	DRC	224	56	
	H5	DRC	86	21	
	H12	DRC	161	40	
	H13	DRC	161	40	
	H17	DRC	972	218	
	P3	BRP	4983	1246	
	P17	BRP	1257	314	
	P20	BRP	230	58	
	P25	BRP	224	56	

DRC = Deck ReConstruction
BRP = Bridge RePlacement

Table 3.4 Output of Optimization Program (40% Needed Budget)

YEAR	BRIDGE No.	ACTIVITY	FED. COST ($1000)	STATE COST ($1000)	TOTAL ($1000)
1	H1	DRC	188	47	3295
	H2	DRC	221	51	
	H6	DRC	109	12	
	H11	DRC	242	27	
	H15	DRC	67	7	
	H20	DRC	95	24	
	H21	DRC	198	49	
	H22	DRC	148	16	
	P1	BRP	400	100	
	P10	BRP	508	508	
	P15	BRP	154	154	
	P23	BRP	336	336	
	P24	BRP	52	52	
2	H16	DRC	139	139	3528
	H18	DRC	53	13	
	H24	DRC	79	20	
	P5	BRP	968	242	
	P14	BRP	236	59	
	P18	BRP	823	206	
	P19	BRP	310	78	
	P20	BRP	230	58	
3	H14	DRC	428	48	3543
	P8	BRP	436	109	
	P9	BRP	1037	259	
	P12	BRP	672	168	
	P16	BRP	308	77	
4	H19	DRC	99	25	3557
	P13	BRP	2522	631	
	P25	BRP	224	56	
5	H3	DRC	310	77	3568
	H4	DRC	224	56	
	H5	DRC	86	21	
	H7	DRC	189	21	
	H8	DRC	1040	260	
	H9	DRC	205	51	
	H10	DRC	297	33	
	H12	DRC	161	40	
	H13	DRC	161	40	
	H23	DRC	237	59	

DRC = Deck ReConstruction
BRP = Bridge RePlacement

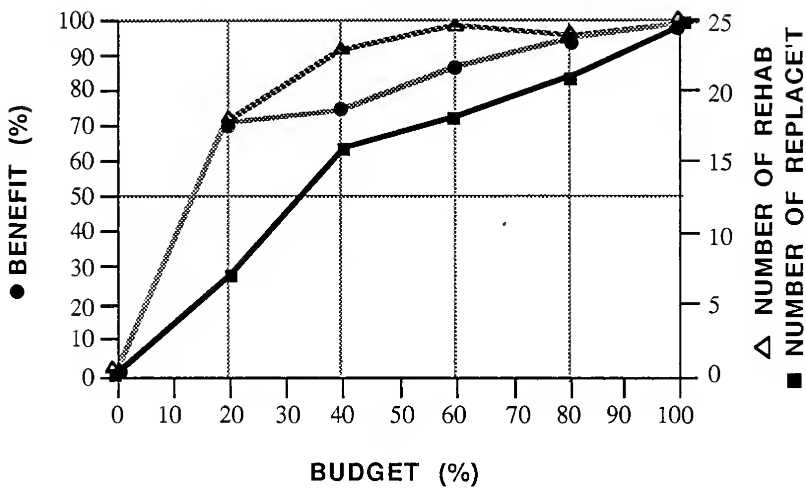

Figure 3.6 Comparison of Optimization Results

these projects are less expensive and more projects can be accommodated to maximize system effectiveness. It can also be seen that the benefit does not decrease as quickly as budget goes down. This phenomenon indicates that the optimization model always attempts to select projects so that the system benefit is as large as possible.

The use of dynamic programming in combination with integer programming and the Markov chain provides bridge managers an optimization tool for managing bridge systems. The model selects projects by maximizing the effectiveness of entire system over a given program period subject to budget constraints. Therefore, for any available budget, the model always gives a mix of projects that maximizes the system effectiveness for the given budget. That is, the model always offers optimal solutions. The priority ranking methods as used in some bridge management systems, however, usually do not guarantee optimal solutions because they are based solely on the comparison of rankings. In a ranking procedure the following two important ingredients may be missing [Cook and Lytton 1987]:

1. evaluation of inter-project tradeoffs in selecting projects,
2. selection of optimal strategies which are guaranteed to adhere to existing budget limitations.

The principle of optimality assures that dynamic programming results in not only the optimal solution for the program period T, but also for any period less than T. These optimal solutions for the subperiods are of importance to bridge managers in scheduling bridge activities. Furthermore, these solutions are also guaranteed by the principle of optimality to be absolute optima rather than

relative optima.

The optimization model has a powerful capability of handling a system with hundreds of bridges. It can be used by highway programmers to gain maximum return by effectively allocating the limited bridge budgets in both short-term and long-term planning horizons.

CHAPTER 4: ALTERNATIVE VERSION OF THE OPTIMIZATION MODEL

4.1 Introduction

Ranking and optimization are two of the most widely used techniques applied in highway project selections. However, these two approaches are very different in concepts. Ranking techniques evaluate several related factors of a project simultaneously and yield a quantitative ranking value based on the evaluation on these factors. Thus, all the considered projects are ranked according to their corresponding ranking values. The ranking methods do not necessarily give an optimal solution. Nevertheless, a ranking approach is simple to use and provides the relative order of importance of different projects. Such an ordered list can be used for decision-makers to make final decisions on the basis of project ranking values. On the other hand, an optimization technique produces an "optimal" solution of a highway system while the projects are selected subject to a set of constraints. The optimal solution is obtained either by maximizing the system benefit or by minimizing the total negative effect on the system that is caused by undertaking the selected projects. Different from ranking methods, optimization techniques do not follow the rule of "choosing projects with the worst conditions", instead, the optimization techniques select projects that contribute the most benefit to the highway system while all of the constraints are satisfied simultaneously.

Like many other pavement and bridge management systems, the Indiana Bridge Management System (IBMS) provides two separate procedures, ranking and optimization models, for selecting bridge rehabilitation and replacement projects. The ranking model has been discussed in Volume 5 of this report and

the optimization model has been presented in Chapter 3 of this volume. The ranking model considered several evaluation criteria in terms of utility functions, while the optimization model used a direct representation of the evaluation criteria. Because of the different concepts of the two techniques, the two models would produce two different sets of results. Consequently, it was felt to be desirable to combine the techniques so that the ranking and optimization models would have the same form of representation of evaluation criteria and the results could be compared and analyzed according to a common basis.

This chapter presents the approach used to combine ranking and optimization techniques in one model with two phases. In this combined model, the ranking values are first computed on the basis of appropriate utility functions. If the decision-maker wants to use only the ranking information, the model provides a list of prioritized projects and stops. On the other hand, if an optimized list of actions is desired over a period under given scenarios of funding, the model continues with the utility values generated in the first phase and provides the results by maximizing the systemwide gain in utility values.

4.2 <u>The Ranking Model</u>

Setting priorities on pavement and bridge related projects is usually a multi-attribute decision-making problem, requiring decision-makers to evaluate simultaneously several related factors. The ranking model of IBMS was developed using the technique of the analytic hierarchy process (AHP) [Saaty 1980]. The AHP method is a useful tool to rank projects when subjective judgments are involved. However, a direct application of the method may not be practical when the number of projects is large. For example, even when there are only 22 bridge

projects to compare, one has to make 231 pairwise comparisons for each evaluation criterion (22(22-1)/2). Assuming there are six criteria under consideration, the number of pairwise comparisons goes up to 1,386. In reality, the number of projects may range between 500 and 1000, and the direct use of the AHP is thus, impractical.

The above problem, however, can be solved by the inclusion of the concept of utility. In a highway facility management system, utility is the level of overall effectiveness that can be achieved by undertaking a project. If an appropriate utility is assigned to projects with respect to certain evaluation criteria, the expected utility of each alternative project can be evaluated. Then, the top priority project is the alternative with the highest expected utility value. In the current version of the ranking model, utility functions were included for the following evaluation factors: average daily traffic volume, estimated remaining service life, structural condition rating, bridge traffic safety index, and community impact index in terms of detour length.

4.3 The Optimization Model

Optimization techniques are used to obtain a list of projects so that an objective function, such as systemwide condition, or level of service, can be optimized subject to a set of budget and other constraints over time. For the Indiana Bridge Management System, such a model was developed on the basis of dynamic programming and integer programming, as discussed in Chapter 3 of this volume. Markov chain transition probabilities of bridge structural conditions were used in the model to predict or update bridge structural conditions at each stage of the dynamic programming.

In the optimization model, the primary measure of effectiveness was the systemwide improvement in bridge structural condition. All evaluation factors used in the ranking model were also included. However, their representation was not in the form of utility functions. Instead, they were incorporated in the objective function as weighing factors to the primary measure of effectiveness, as shown in Equation 3.4.

4.4 Combined Model

Any of the two models for project selection, either ranking or optimization, can be used to select bridge projects based on priority order or optimization with respect to systemwide benefit. The ranking model for IBMS uses utility functions for each of the evaluation factors considered. The inclusion of these factors in the process of bridge project selection makes the associated utility values reflect the main concerns of bridge rehabilitation and replacement activities. The utility values produced by the ranking model were thus used to formulate the objective function of the optimization model.

With utility values as common measures for the ranking and optimization models, the combined model was developed by modifying the existing models. Because the utility values range from 0 to 100, with 0 being the utility value of a "perfect" bridge and 100 the value of the "worst" bridge, the utility value of a bridge will decrease after undertaking a rehabilitation activity. Thus, the difference between utility values of before and after undertaking a bridge activity would indicate the improvement in overall utility. This difference, therefore, was defined as the effectiveness or benefit of the bridge activity. Incorporating this definition into the dynamic optimization model, the objective

function was to maximize the total decrease of utility values of the bridge system subject to the budget constraints.

In order to combine the two models, the only modification of the dynamic programming formulation was to change Equation 3.4 to the following:

$$E_i = U_{ib} - U_{ia} \tag{4.1}$$

where:

E_i = effectiveness gained by bridge i if an activity is undertaken;

U_{ib} = utility value of bridge i before the activity is undertaken;

U_{ia} = utility value of bridge i after the activity is undertaken.

Thus, the formulation of the new approach is obtained by substituting Equation 4.1 for Equation 3.4, while Equations 3.5 through 3.15 would remain unchanged. The value of E_i would be available from the ranking model. This value is the weighted summation of individual utility differentials for economic efficiency, remaining service life, structural condition, traffic safety and community impact.

The change of Equation 3.4 to Equation 4.1 combines the ranking model and the optimization model. Thus, the result obtained from the optimization model could be directly compared with that of the ranking model in terms of the total gain in utility value changes. The change in the computation was to have Equation 4.1 as a subroutine of the dynamic optimization program. This subroutine is, in effect, the ranking program. At each stage of the dynamic

optimization process, the ranking program, as a subprogram, would compute the system benefit, or the total gain of utility value changes, and the dynamic programming as the main program would make optimal project selection according to the systemwide benefit.

4.5 An Application Example

To illustrate the combined model, fifty state highway bridges in Indiana which need rehabilitation or replacement were selected to run the combined model. Table 4.1 presents the results that were obtained from the ranking portion of the model. The bridges in Table 4.1 are listed in the order of priorities so that one can select rehabilitation projects from the top of the list.

Since the project selection in the combined approach depends on available budgets, the optimization program was run several times using different given budgets. The results of one of the runs are shown in Table 4.2. It should be noted that the bridges in Table 4.2 are not presented in a priority list as those in Table 4.1. This run was made with a given budget of $11,128,000, or about 25% of the total budget needed for repairing and replacing all of the 50 bridges. The total gain in utility, or the systemwide benefit, was 900.0. With the same amount of budget, one can also select bridge projects from the ranking list in Table 4.1. Selecting the bridges from top of the list, the first six bridges in Table 4.1 could be chosen with the given budget. Thus, with this selection the total cost is $10,081,180, and the total gain of the utility is 272.6.

By dividing the total gain of utility by its corresponding total cost, the

Table 4.1 Output of the Ranking Model

Bridge No.	Priority	U_i	E_i	ΣE_i	C_i	ΣC_i	Activity
31	1	72.9	52	52	2159	2159	BRP
30	2	72.5	50	102	1210	3369	BRP
47	3	72.3	46	148	1549	4918	BRP
27	4	70.4	53	201	5000	9918	BRP
49	5	69.9	51	252	65	9983	DRC
24	6	69.0	21	273	98	10081	DRC
25	7	68.9	18	291	1993	12074	DRC
26	8	68.0	52	343	500	12574	BRP
46	9	67.6	50	393	965	13539	BRP
33	10	65.0	50	443	545	14084	BRP
50	11	65.0	50	483	280	14364	BRP
28	12	63.2	48	541	6228	20593	BRP
37	13	61.2	50	592	840	21433	BRP
48	14	60.5	50	642	420	21853	BRP
32	15	60.1	51	693	3409	25262	BRP
42	16	60.1	50	743	1571	26833	BRP
40	17	59.4	50	793	193	27026	BRP
17	18	59.0	46	839	1090	28116	DRC
43	19	59.0	50	889	1029	29145	BRP
44	20	59.0	50	939	388	29533	BRP
45	21	59.0	50	989	288	29821	BRP
23	22	55.7	40	1029	296	30117	DRC
35	23	52.7	42	1071	635	30759	BRP
34	24	51.7	42	1113	1297	32049	BRP
10	25	51.7	29	1142	330	32379	DRC
38	26	50.0	41	1183	3153	35532	BRP
36	27	49.2	38	1221	840	36372	BRP
39	28	46.0	37	1258	295	36667	BRP
11	29	42.0	15	1273	269	36936	DRC
29	30	42.0	33	1306	2192	39128	BRP
13	31	36.0	23	1329	201	39329	DRC
22	32	36.0	13	1342	164	39493	DRC
41	33	35.9	29	1372	385	39878	BRP
9	34	32.6	15	1387	257	40135	DRC
12	35	32.0	19	1406	201	40336	DRC
21	36	31.9	9	1415	247	40583	DRC
8	37	30.3	15	1430	1300	41883	DRC
18	38	28.6	8	1438	66	41949	DRC
7	39	28.4	12	1450	210	42159	DRC
3	40	28.2	12	1462	387	42546	DRC
6	41	27.0	10	1472	121	42668	DRC
20	42	26.9	8	1480	119	42787	DRC
14	43	26.8	4	1484	476	43262	DRC
16	44	26.6	10	1494	154	43416	DRC
15	45	23.0	4	1498	74	43491	DRC
4	46	22.3	8	1506	281	43771	DRC
5	47	21.9	8	1513	107	43878	DRC
1	48	20.4	8	1521	235	44113	DRC
2	49	20.0	8	1529	276	44389	DRC
19	50	19.9	4	1533	124	44512	DRC

U_i = Utility Value of Bridge i.
E_i = Effectiveness of Bridge i.
C_i = Cost of the Activity of Bridge i, in \$1000.
BRP = Bridge Replacement.
DRC = Deck Reconstruction.

Table 4.2 Output of the Proposed Approach

Available Budget = $11,128,000
Total Objective Value (Benefit) = 900.0

Bridge No.	E_i	C_i	Activity
1	8	235	DRC
4	8	281	DRC
6	10	121	DRC
7	12	210	DRC
9	15	257	DRC
10	29	330	DRC
11	15	269	DRC
12	19	201	DRC
13	23	201	DRC
15	4	74	DRC
16	10	154	DRC
17	46	1090	DRC
18	8	66	DRC
19	4	124	DRC
20	8	119	DRC
21	9	247	DRC
22	13	164	DRC
23	40	296	DRC
24	21	98	DRC
26	52	500	BRP
33	50	545	BRP
35	42	635	BRP
36	38	840	BRP
37	50	840	BRP
39	37	295	BRP
40	50	193	BRP
41	29	385	BRP
44	50	388	BRP
45	50	288	BRP
46	50	965	BRP
48	50	420	BRP
50	50	280	BRP

E_i = Effectiveness, or Change of Utility Value, of Bridge i.
C_i = Cost of the Activity of Bridge i, in $1000.
BRP = Bridge Replacement.
DRC = Deck Reconstruction.

gain of utility per million dollar for the proposed approach is:

$$\frac{9000.0\ (utility)}{11128000\ (\$)} = 81\ units\ per\ million\ dollars,$$

and that for the ranking method is:

$$\frac{272.6\ (utility)}{10081180\ (\$)} = 27\ units\ per\ millon\ dollars.$$

Therefore, the value of the proposed approach is three times as large as the value of the ranking method in this example.

Figure 4.1 is a comparison of the results from the two approaches in terms of system benefits and available budget. It can be seen that the optimization approach always gives better solution than the ranking approach when the available funds are less than 100% of the need.

4.6 Chapter Conclusions

By defining the system benefit as the total gain in utility value changes, the ranking and optimization models were combined into one model. Through an example of fifty bridge projects the usefulness of the utility based optimization approach was demonstrated. In bridge management systems, both ranking and optimization techniques are used for project selection. However, an optimization approach would insure an optimal use of resources.

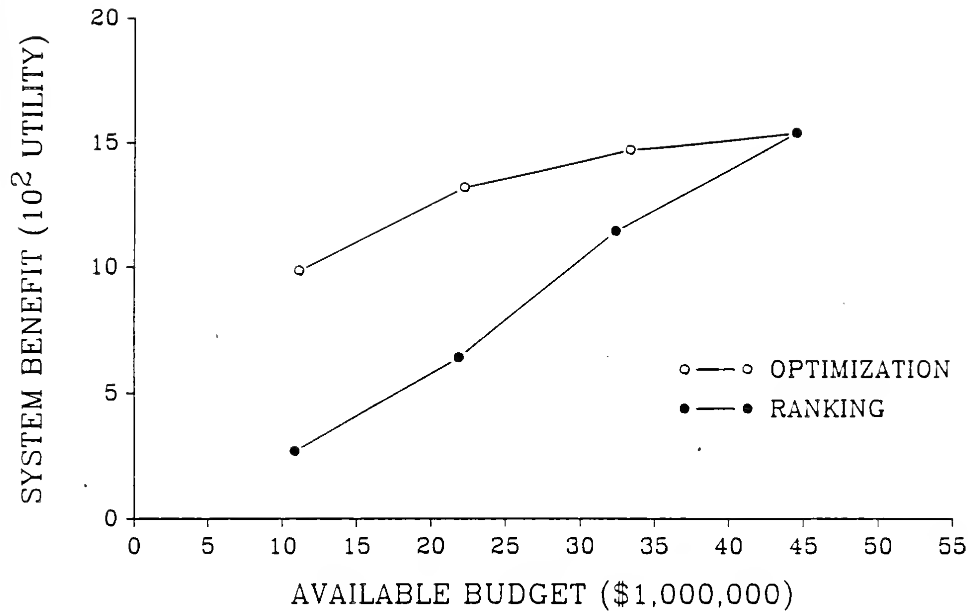

Figure 4.1 Comparison of Project Selections by Ranking and Optimization Approaches

CHAPTER 5: SUMMARY AND CONCLUSIONS

This volume presents the results of a study to develop a bridge condition prediction model and a dynamic optimization model for the Indiana Bridge Management System. The techniques applied in this study, such as Markov chain and the combination of dynamic programming and integer linear programming, are new in the field of bridge management. Both the bridge condition prediction model and the optimization model considered the stochastic and dynamic nature of bridge condition changes, and therefore, the models reflected better the reality of bridges, and therefore, the models reflected more closely the real world bridge performance than the tradition deterministic approaches.

Based on the findings of this research, the following conclusions can be made:

1. The bridge condition prediction model can be used to predict the future condition rating of a bridge as well as to predict the average condition rating of a group of bridges.

2. The performance functions can be used as a measure of effectiveness of bridge activities.

3. The dynamic optimization model is an efficient tool for bridge project selection at the network level. It can be used by decision makers to gain maximum return by effectively spending the limited bridge budgets

in both short-term and long-term planning horizons.

4. The dynamic optimization model can also be used for sensitivity analysis or bridge system planning by inputting different budget levels.

5. A dynamic optimization model based on utility values generated by the ranking approach is an improved decision-making tools.

6. It is a limitation that the current version of the dynamic optimization model can work only on main frame computers, but not on microcomputers due to its complexity and large computer space requirements.

It should be noted that the reliability of the results of a model depends on the accuracy of the input data. Therefore, the importance of data reliability, uniformity, and consistency must be always emphasized. The research done in this study on condition assessment, traffic safety evaluation, cost analysis, timing of bridge improvement activities, and other elements of the bridge management system was all directed to make the needed input information to project selection precise and reliable.

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